1. Series or sequence?

I consider myself as more of a series.

2. How are we supposed to show computations in words, it is pretty straight forward with rules?

Narrating what you are doing is one way to do it. However, I am really looking for you to tell me what you are thinking. Certainly you have to stop and think about what you are doing and you must get stuck, even momentarily. I want to know when this happens and I want you to recognize when this happens. I also want you to question basic mathematical rules that you use and reassure yourself that they apply to your current situation.

3. Is the textbook a good place to start reviewing?

Yes, each section has dozens and dozens of problems for you to work on. The end of each chapter also has practice tests. If you don’t understand a section there are also several examples in each section that are worked out in detail.

4. Will there be a curve not he final grade?

I do not think so because the midterm and the third exam were both curved.

5. What is the average on exam 3?

The average for our sections (C1, D1 and E1) was 62 with a median of 63. The other sections averaged 57 with a median of 54.

6. Can you explain Taylor series?

Like I talked about in class Taylor series allow us to convert any differentiable function into a polynomial. Polynomials are really easy to compute, I believe I showed that to get a good approximation for \( \sin(1) \) you can use three terms of it’s Taylor series expansion (centered at \( a = 0 \) ) and be quite close: \( \sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \)

so \( \sin(1) \approx 1 - 1/6 + 1/120 = .8416 \) and the real answer is .84147098....

In general, if \( f(x) \) is infinitely differentiable function (most functions you have ever seen are infinitely differentiable) then the Taylor series expansion about the point \( a \) is

\[
\frac{f(a)}{0!} + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} + \frac{f^{(4)}(a)(x-a)^4}{4!} + \frac{f^{(5)}(a)(x-a)^5}{5!} + \cdots
\]

You may also be asked to produce the first order, second order, third order, etc. Taylor polynomial, which basically just stops the Taylor expansion at the first, second, third, etc. power of \( x \).

For example, let’s find the Taylor series expansion for \( f(x) = \sin(x) \) centered at \( a = 0 \). What I usually do is make a table with the derivatives and the derivatives evaluated at \( a \):

| \( f(x) = \sin(x) \) | \( f(0) = \sin(0) = 0 \) |
| \( f'(x) = \cos(x) \) | \( f'(0) = \cos(0) = 1 \) |
| \( f''(x) = -\sin(x) \) | \( f''(0) = -\sin(0) = 0 \) |
| \( f'''(x) = -\cos(x) \) | \( f'''(0) = -\cos(0) = -1 \) |
| \( f^{(4)}(x) = \sin(x) \) | \( f^{(4)}(0) = \sin(0) = 0 \) |
| \( f^{(5)}(x) = \cos(x) \) | \( f^{(5)}(0) = \cos(0) = 1 \) |
Now we can start putting our data into the Taylor expansion.

\[ P(x) = \frac{0}{0!} + \frac{1(x - 0)^1}{1!} + \frac{0 \cdot (x - 0)^2}{2!} - \frac{1(x - 0)^3}{3!} + \frac{0(x - 0)^4}{4!} + \frac{1(x - 0)^5}{5!} - \cdots = \]

\[ x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots . \]

Note that the first and second order Taylor polynomial are the same, as are the third and fourth, the fifth and sixth, etc.

If we instead center the Taylor expansion around \( a = 10 \) (Note: the closer the \( x \) value that you are approximating is to \( a \) the better the approximation i.e. if you center your expansion about \( a = 0 \) you should use it to approximate numbers pretty close to 0. If you expand around \( a = 10 \) use it to approximate numbers pretty close to 10) we get

\[ P(x) = \frac{\cos(10)(x - 10)}{1!} - \frac{\cos(10)(x - 10)^3}{3!} + \frac{\cos(10)(x - 10)^5}{5!} - \frac{\cos(10)(x - 10)^7}{7!} + \cdots . \]

7. What is the radius of convergence?

The radius of convergence helps us determine for which values a series will converge. If you choose numbers inside of the radius of convergence the series will converge, if you choose numbers outside the radius of convergence the series will diverge. Check out the first three problems of the worksheet.

8. What is the difference between Taylor series and MacLauren series?

The Taylor series is explained above. The MacLauren series is the Taylor series when \( a = 0 \).

9. What are your thoughts about VEISHA?

I think it should continue. I think the calculation affected those who were not being irresponsible more than those who were being irresponsible. Also, drunk people are dumb.

10. Who is in charge of naming new mathematical concepts?

Usually the mathematician that invents the idea gets to name it. For example, I came up with the term positive semidefinite migration. However, sometimes rich people get to name things like L’Hospital. L’Hospital was a rich dude who was pretty good at math but he hired someone who was REALLY good at math named Bernoulli to tutor him. Everyone knows that it was Bernoulli who came up with L’Hospital’s Rule of limits.

11. What is the difference between \( \frac{1}{1-x} \) and \( \frac{1}{1+x} \)?

\[ \frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots \text{ for } |x| < 1. \]

\[ \frac{1}{1-x} = 1 - x + x^2 - x^3 + x^4 - \cdots \text{ for } |x| < 1. \]

12. What programming languages do you know? How experienced are you?

I mostly use SAGE which is python based. I took a few classes of C, I got up to object oriented programming. Most of my algorithms are brute force (which works fine for my purposes) but I am starting to become much more concerned with memory use and efficiency because some of the computations that I will be running will take months or years if I don’t make them better.

13. Explain Einstein’s theory of special relativity in ten words or less using Morse code

...- . .-. -.– - .... .. -. –. / .. ... / ... ... / ... ... ... - - ....
14. What kind of research are mathematicians working on right now?

Mathematics has become such an enormous subject that I would not even know where to start. Some of the big divisions are: Foundations, Discrete math/Algebra, Analysis, Geometry and Topology, Applied math. Each of these can be broken down into 15 -30 sub topics which can be broken down even further. I would bet that dozens (and maybe hundreds) of math papers are published everyday!

15. What is your dream job?
Math professor.

16. Why don’t other recitations do write-ups?
Because their objectives are different than mine. I want to teach you math, work on your communication and get you to think deeper in general. I hope the fact that your test scores are way above the average convinces you that it is a good idea.

17. How long have you been in college (undergrad and grad)?
Too long, I took six years to get my undergrad degree and this is my fifth year as a graduate.

18. Who graded the exam?
I graded the first three problems, I think Maksym graded the second three and Xavier graded the last two.

19. Is there anything else to be done in the world of math?
Yes, there are probably tens of thousands of problems whose answer is unknown. For example, can every even integer greater than 2 be expressed as the sum of two primes (2 + 2 = 4, 3 + 3 = 6, 3+ 5 = 8, 5+5 = 10, ...).

20. On the root test, why does the series converge when \( p < 1 \)?

The idea is that if \( p < 1 \) then the series you are looking at is bounded above by a geometric series and you will always be able to squeeze the geometric series in between \( p \) and 1 so the geometric series will converge, thus your original series will converge.

21. How do you get the first 6 terms of \( \sin(x) \cos(x) \)?

You do infinite polynomial multiplication which is basically like when you FOIL two linear functions. So what I do is look at the expansions:

\[
\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \cdots
\]

\[
\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \cdots
\]

Now we have two options. You can ask “What are the ways I can multiply two terms together to get an \( x^1 \) term? The answer is you can multiply the first two terms and that is it. Now you ask, "What are the ways I can multiply two terms together to get an \( x^2 \) term?" The answer is you cannot do this by choosing one term from each series. Now you ask, "What are the ways I can multiply two terms together to get an \( x^3 \) term?"

There are two ways, you can do \( x \cdot \left( -\frac{x^3}{2!} \right) \) or you can do \( 1 \cdot \left( -\frac{x^3}{3!} \right) \). Continue in this process until you have come up with 6 terms.

The other way is to multiply ‘everything’ in the \( \sin(x) \) expansion by 1 and then everything by \( -\frac{x^2}{2!} \) and then everything by \( \frac{x^4}{4!} \) and then combine the terms that have the same powers of \( x \).