1. Do you want questions on a separate piece of paper?

Yes, if your questions are not on a separate piece of paper you will not get credit for attending class. You must also put the section and date on the questions.

2. What is the physical interpretation of time?

What physicists refer to time is basically the ticking of a very accurate clock. A grandfather clock measures time by a pendulum swinging back and forth and each swing corresponds to a second, although this is not very accurate. A second used to be defined based on how long it took the earth to revolve around the sun, and this actually was pretty accurate. Now a second corresponds to how many times a Cesium atom, at 0 degree Kelvin, transitions between two ground states. You can think of each transition as a pendulum swinging back and forth except this pendulum swings back and forth 9,192,631,770 times every second. These are called atomic clocks, one of them is in Switzerland and expected to only be off by one second over the next 30 million years. Interestingly, atomic clocks can also be used to show that time is relative. Note that this implies we can travel forward in time but not backward.

3. Is mathematics useful to study engineering?

Yes, in all aspects of engineering (and other quantitative disciplines), when confronted with a problem, the usual course of events is to define a model using mathematical equations describing the relationships of the different aspects of the problem. Usually, these relationships are described using calculus.

Examples of basic things that occur all the time in Engineering are rates of change with respect to time, or space of such variables as heat, wave, gas, electric current, electromagnetic fields, conductivity (of heat, current, etc. in various materials), vibrations in solids like bridges and buildings, and many others.

The basic ones that a Freshman studies involve problems that seek to maximize or minimize a quantity (such cost or profit, or a surface area for some object, or the distance a projectile can achieve). You’ll also find calculus in Probability and that in turn is used in engineering when you can’t get a model for your problem but instead you have many data items from which you extract relashionships .

These problems use calculus (derivatives and integrals) to be formulated and then solved either exactly (called a closed form solution) or numerically (approximate solution).

There is so much more, but for now, you get the breadth and scope for Calculus in Engineering.

4. Do we cover Taylor and McLauren series this semester?

I believe we do, it is probably the most applicable part of calculus so get ready.

5. What team?

El Heat.
6. What is \( \int_1^\infty \frac{1}{\sqrt{1 + x^2}} \, dx \).

Ok, I think I finally got this one. There are a few things going on here. We are actually going to have to do two substitutions for this one. Also, when I do the substitutions I am not going to change my limits of integration, I am going to use a \( u \) until I get back to the original variable \( x \). The first substitution I will use is \( x = \tan(u) \) (because then \( \sqrt{1 + \tan^2(u)} = \sec(u) = \sec(u) \)) and when I differentiate I have \( dx = \sec^2(u) \, du \). Substituting we have:

\[
\int_1^\infty \frac{1}{\sqrt{1 + x^2}} \, dx = \int_\ast^\infty \frac{\sec^2(u) \, du}{\sqrt{1 + \tan^2(u)}} = \int_\ast^\infty \sec(u) \, du.
\]

Now we let \( v = \sec(u) + \tan(u) \) so \( dv = \sec(u) \tan(u) + \sec^2(u) \, dv = \sec(u)(\sec(u) + \tan(u)) \, du \). Now we substitute and get

\[
\int_\ast^\infty \sec(u) \, du = \int_\ast^\infty \frac{dv}{\sec(u)(\sec(u) + \tan(u))} = \int_\ast^\infty \frac{dv}{\sec(u) + \tan(u)} = \int_\ast^\infty \frac{dv}{v} = \ln|v|^{\ast}_\ast.
\]

Now we resubstitute to get back to our original variable \( x \). Note that we have to construct a reference triangle to figure out \( \sec(u) \) in terms of \( x \).

\[
\ln|v|^{\ast}_\ast = \ln|\sec(u) + \tan(u)|^{\ast}_\ast = \ln|\sqrt{1 + x^2} + x|^{\ast}_1.\]

Now we introduce our variable to handle the infinity

\[
\ln|\sqrt{1 + x^2} + x|^{\ast}_1 = \lim_{b \to \infty} \ln|\sqrt{1 + x^2} + x|^{b}_1 = \lim_{b \to \infty} \ln|1 + b^2 + b| - \ln|\sqrt{2} + 1| = \infty.
\]

That means this integral DIVERGES, in this case it keeps getting bigger and bigger and bigger and bigger and bigger and ....

7. What did the fox say?

werkasswdjlwr poajwerjw wweorj werja fsadlkjf wer

8. How do you find whether two functions converge? What does it mean if we say that a function converges?

To find out whether or not two functions converge we mean do they ever end up looking really similar to one another when the \( x \) values get really big. So if you think about an on ramp onto a highway you can think of the highway as one function and the ramp as another function. As you move along the on ramp (as \( x \) gets bigger) the ramp and the highway start to look like the same road (function), we would say that these two 'functions' converge. In math we use limits to see what happens to functions when \( x \) gets really big. For example, consider the functions \( \frac{x + 1}{x} \) and \( \frac{e^x - 1}{e^x} \). If you graph these functions they don’t really look like the same thing but when you take \( \lim_{x \to \infty} \) of both of them they do end up being the same thing.

9. What is in your cup of coffee?

Melange

10. What can run but never walks, has a mouth but never talks, has a head but never weeps, has a bed but never sleeps?

A river.
11. Find \( \int_0^2 \frac{dx}{1-x} \).

Here we notice that our limits of integration, which correspond to \( x \) values, range from 0 to 2. However, if we set the denominator of our function equal to zero we have \( 1-x=0 \Rightarrow x=1 \). This means that when \( x=1 \) we end up dividing by zero WHICH IS A VERY BAD THING TO DO!!! This means we have to split up our integral into two pieces and we split it right where our bad spot is, \( x=1 \). So we get

\[
\int_0^2 \frac{dx}{1-x} = \int_0^1 \frac{1}{1-x} \, dx + \int_1^2 \frac{1}{1-x} \, dx.
\]

But notice that we will still end up dividing by zero because our limits of integration still contain 1. To get around this we use limits. For the first integral our \( x \) values are always be smaller than 1 and for the second integral our \( x \) values will always bigger than 1 so we have to use left and right limits.

\[
\lim_{b \to 1^-} \int_0^b \frac{1}{1-x} \, dx + \lim_{b \to 1^+} \int_b^2 \frac{1}{1-x} \, dx = \lim_{b \to 1^-} \ln |1-x| \bigg|_0^b + \lim_{b \to 1^+} \ln |1-x| \bigg|_b^2 = \\
\lim_{b \to 1^-} \ln |1-b| - \ln |1-0| + \lim_{b \to 1^+} \ln |1-2| - \ln |1-b| = \lim_{b \to 1^-} \ln |1-b| - \lim_{b \to 1^+} \ln |1-b|
\]

because \( \ln(1) = 0 \).

Now we have to do some thinking (I actually have been thinking about this for a while now). What we notice is that in \( \lim_{b \to 1^+} \ln |1-b| \) the \( 1-b \) is always going to be negative but then the absolute value turns it into a positive. So we can actually rewrite it as \( \lim_{b \to 1^-} \ln |1-b| = \lim_{b \to 1^-} \ln |1-b| \) and we will be safe, think about this for a while, I had to. Then we get

\[
\lim_{b \to 1^-} \ln |1-b| - \lim_{b \to 1^+} \ln |1-b| = \lim_{b \to 1^-} \ln |1-b| - \lim_{b \to 1^-} \ln |1-b| = \lim_{b \to 1^-} \ln |1-b| - \ln |1-b| = 0.
\]

12. Will you drop the least score of one of the exams?

It is not up to me but no.

13. Applications of math in computer science?

Graph theory is an advanced math topic that is extremely useful when studying computer science. Discrete math is also used in the study of computer science. Figuring out how efficient your algorithms are and whether or not your algorithms do what you want them to do is another application. If we want to get more basic the binary number system, modular arithmetic and number theory are some other applications.

14. How old is your dog?

I have two dogs, they are 4 and 5 years old.
15. What are the two types of improper integration?

The two types of improper integration are integrals whose limits are infinite. For example \( \int_{-\infty}^{2} x^2 \, dx \), \( \int_{-\infty}^{\infty} 1 \, dx \), or something similar. The other type is when the function has a vertical asymptote that is included in the range of your limits of integration. For example \( \int_{1}^{1} 1 \, dx \). The function \( \frac{1}{x} \) has a vertical asymptote at \( x = 0 \) which is within our limits of integration \([-1, 1]\). However, just because a function has a vertical asymptote doesn’t mean you have to worry about it. Here \( \int_{-2}^{3} \frac{1}{(x-1)(x+4)(x+3)} \, dx \) we only have to worry when \( x = 1 \), we can ignore the other two vertical asymptotes.

16. What are the things we’re supposed to know, similar to \( \ln(0) = -\infty \)?

I would say you should know how to find the \( x \) and \( y \) intercepts of almost any function. You should also know the general shape of \( e^x \), \( \ln(x) \), \( 1/x \), \( 1/x^2 \), \( |x| \), \( x^2 \), \( x^3 \) and the trig functions. Also being able to figure out about what a rational function looks like, for example \( \frac{x+1}{x-3} \) has a vertical asymptote at \( x = 3 \), a horizontal asymptote at \( y = 1 \), y intercept of \(-1/3\) and \( x \) intercept of \( x = -1 \). Being able to transform functions is really helpful too. For example, \( \ln(x+2) \) is the \( \ln(x) \) function shifted to the left 2 units, \( e^{x-2} \) is the \( e^x \) function shifter right two units and \(-\sqrt{x+2} \) is the \( x^2 \) function shifted to the left two units and then reflected over the \( x \) axis.

17. How should I review for the next exam?

That is a great question. Certainly be able to do all of the homeworks between the midterm and the next exam. Redo any quizzes that were given, and when I say redo I mean sit down for 10 minutes, no notes, no calculator and take the quiz. In fact, something that I used to do is make up my own exam and then take the exam under the same circumstances that the real exam will be in. I think too often you end up doing your homework or write-ups in a relatively low stress environment and not much as far as time constraints are concerned. This does not prepare you for the exam very well. Also, if you cannot think of what types of questions would appear on the exam then you are certainly not ready for the real exam. Also, COME TO MY OFFICE HOURS IF YOU DON’T UNDERSTAND SOMETHING, I MAY BE YOUR BEST RESOURCE FOR THIS CLASS.

18. \( \int_{-\pi/2}^{\pi/2} \frac{\tan^2(x/2)}{\sec^3 x/2} \, dx \).

I don’t have time right now but first I would do a substitution \( u = x/2 \), it will makes everything easier. Then I would convert everything to \( \sin(x) \) and \( \cos(x) \). That is a good start.
19. \( \int_{-\infty}^{\infty} \frac{1}{x^2} \, dx \).

Notice that this integral is doubly improper because it has a vertical asymptote and the limits of integration are infinite. First we split at the vertical asymptote of \( x = 0 \):

\[
\int_{-\infty}^{\infty} \frac{1}{x^2} \, dx = \int_{-\infty}^{0} x^{-2} \, dx + \int_{0}^{\infty} x^{-2} \, dx.
\]

We are going to work on one integral at a time. There is some thinking to be done at this point, notice that for the first integral we need the bottom limit to go to \(-\infty\) and the top limit to go to 0 from the left. This can be accomplished with one limit:

\[
\int_{-\infty}^{0} x^{-2} \, dx = \lim_{b \to -\infty} \int_{b}^{1/b} x^{-2} \, dx = \lim_{b \to -\infty} -\frac{1}{x} \bigg|_{b}^{1/b} = \lim_{b \to -\infty} -\frac{1}{b} + \frac{1}{b} = \infty.
\]

Make sure you understand what just happened because we are about to do the same thing to the other integral.

\[
\int_{0}^{\infty} x^{-2} \, dx = \lim_{b \to \infty} \int_{1/b}^{b} x^{-2} \, dx = \lim_{b \to \infty} -\frac{1}{x} \bigg|_{1/b}^{b} = \lim_{b \to \infty} -\frac{1}{b} + b = \infty.
\]

Then we add up the two integrals and we get \( \infty \). The easier way is to notice that \( \frac{1}{x^2} \) is even (it looks the same if you reflect it over the \( y \) axis, so \( \int_{-\infty}^{\infty} \frac{1}{x^2} \, dx = 2 \int_{0}^{\infty} x^{-2} \, dx \).

20. \( \int_{1}^{\infty} \ln(x) \, dx \).

Here we don’t have any vertical asymptotes to worry about which is nice. However, we do have to use integration by parts which is a little tricky. We let \( u = \ln(x) \) so \( du = \frac{1}{x} \, dx \) and \( dv = dx \) so \( v = x \). Now we apply the formula and get

\[
\int_{1}^{\infty} \ln(x) \, dx = x \ln(x) \bigg|_{1}^{\infty} - \int_{1}^{\infty} \frac{1}{x} \, dx = x \ln(x) - x \bigg|_{1}^{\infty}.
\]

Now we introduce our new variable and a limit

\[
x \ln(x) - x \bigg|_{1}^{\infty} = \lim_{b \to \infty} x \ln(x) - x \bigg|_{1}^{b} = \lim_{b \to \infty} b \ln(b) - b - 1 \ln(1) + 1 = \lim_{b \to \infty} b \ln(b) - 1 + 1 = \infty
\]

Notice that \( \lim_{b \to \infty} b \ln(b) - b \) gives us \( \infty - \infty \) which doesn’t mean anything (although you may think it equals 0, STOP THINKING THAT!), but if we change it to \( \lim_{b \to \infty} b \ln(b) - b \) we can see that \( \infty - 1 = \infty \) thus \( \lim_{b \to \infty} b \ln(b) - 1 = \infty \).