1. What ideas and concepts do we cover in Calculus II?

Here is a list of things that we will be investigating throughout the semester.

http://orion.math.iastate.edu/calculus/obj/objectives_Calc_II.html

Much of what we do has applications in Differential Equations which shows up in many Applied Physics contexts; many scientific endeavors involve Applied Physics. Many of the techniques also give us the ability to transform problems into something that is much more manageable. On a side note, Calculus II is not the hardest University Mathematics course as some of you may believe. With hard work you can all do well in this class.

2. What trig identities do I need to know?

One of the goals of the worksheet was to get you to realize that you really only need to memorize a couple of key equations and from these you can derive almost any trig identity that you need. This is difficult at first but with practice it becomes much easier. Of course you are free to memorize the 10 or so trig identities that you will be using in this class (double angle formulas, half angle formulas, additive angle formulas, etc.) The ones on the worksheet should get you through most of the problems though.

3. What is $\int \sin^2(x) \cos^2(x) \, dx$?

I am glad that several people asked this question. Some integrals of the form $\int \sin^n(x) \cos^m(x) \, dx$ can be solved using only trig identities and $u$-substitution. So in theory most of you can do these types of problems already. However, to do this integral requires a technique called integration by parts which we have not learned yet. It also involves many trigonometric identities and $u$-substitution.

4. What is $\int \tan^2(x) \, dx$?

Another great question that illustrates the importance of trigonometry and $u$-substitution. Since those are the only tools required I will show you how to do this problem.

First recall that $\frac{d}{dx} [\tan(x)] = \sec^2(x)$. We can use $\sin^2(x) + \cos^2(x) = 1$ and the following list to derive the necessary trig identities:

- $\sin(x) = \frac{1}{\csc(x)}$
- $\cos(x) = \frac{1}{\sec(x)}$
- $\tan(x) = \frac{\sin(x)}{\cos(x)} = \frac{1}{\cot(x)}$
- $\cot(x) = \frac{1}{\tan(x)} = \frac{\cos(x)}{\sin(x)}$
- $\sec(x) = \frac{1}{\cos(x)}$
- $\csc(x) = \frac{1}{\sin(x)}$
Starting with \( \sin^2(x) + \cos^2(x) = 1 \) we divide every term by \( \cos^2(x) \) to get \( \frac{\sin^2(x)}{\cos^2(x)} + \frac{\cos^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)} \). From above this turns into \( \tan^2(x) + 1 = \sec^2(x) \).

Now we can start integrating.

\[
\int \tan^2(x) \, dx
\]

We let \( u = \tan(x) \) so \( du = \sec^2(x) \, dx \) which implies \( \frac{du}{\sec^2(x)} = dx \). Now we are going to substitute for \( dx \) but not \( u \) and then use \( \tan^2(x) = \sec^2(x) - 1 \) and properties of fractions to get

\[
\int \tan^2(x) \frac{du}{\sec^2(x)} = \int \frac{\sec^2(x) - 1}{\sec^2(x)} \, du = \int \left( \frac{\sec^2(x)}{\sec^2(x)} - \frac{1}{\sec^2(x)} \right) \, du = \int 1 \cdot du - \int \frac{1}{\sec^2(x)} \, du.
\]

Notice that I have been careful not to switch from \( du \) to \( dx \). Also notice that we can integrate the first integral and that the integrand (the thing inside the integral) of the second integral is \( \frac{du}{\sec^2(x)} = dx \). We also resubstitute our \( u = \tan(x) \) which leads us to

\[
\int 1 \cdot du - \int \frac{1}{\sec^2(x)} \, du = u - \int 1 \cdot dx = u - x = \tan(x) - x + C.
\]

Therefore

\[
\int \tan^2(x) \, dx = \tan(x) - x + C.
\]

We can check this solution by seeing if \( \frac{d}{dx}[\tan(x) - x + C] = \tan^2(x) \). I will leave that to you. This is also an example of a good write-up. I do not expect you to type up your solutions but please be very organized and explain what you are doing in words as you go along.

5. What is the integral \( \int x\sqrt{x + 2} \, dx \).  

This is a \( u \)-substitution problem. Let \( u = x + 2 \) then \( du = dx \). Now we substitute for \( x + 2 \) and \( dx \) and get

\[
\int x\sqrt{x + 2} \, dx = \int x\sqrt{u} \, du.
\]

At this point you may think we are in trouble but because \( u = x + 2 \) we can subtract 2 from both sides and get \( u - 2 = x \). Now we make this substitution, and use \( \sqrt{u} = u^{1/2} \), \( \int u^{3/2} \, du = \frac{u^{5/2}}{5/2} + C \) and \( \frac{1}{a/b} = \frac{b}{a} \) to get

\[
\int x\sqrt{u} \, du = \int (u - 2)u^{1/2} \, du = \int u^{3/2} - 2u^{1/2} \, du = \frac{u^{5/2}}{5/2} - \frac{2u^{3/2}}{3/2} + C = \frac{2u^{5/2}}{5} - \frac{4u^{3/2}}{3} + C.
\]