Week 2 write up

\[ y = x^2 \]

\[ r(x) = \sqrt{y} - 3 \]

\[ R(x) = 1 - x^2 = 1 + 3 = 4 \]

Use the washer method to find the volume

\[ V = \pi \int_{a}^{b} (R(x)^2 - r(x)^2) \, dx \]

Since the region is being revolved around the y-axis, \( y = x^2 \) must be written in terms of \( x \) as \( x = \sqrt{y} \)

To find the volume

\[ V = \pi \int_{0}^{1} (4^2 - (\sqrt{y} - 3)^2) \, dx \] \[ \rightarrow \text{expand} \] \[ = \pi \int_{0}^{1} (16 - y + 6\sqrt{y} + 9) \, dx \]

Integrate the function \[ \pi \left( 7y - 4y^{3/2} - \frac{1}{2} y^2 \right) \bigg|_{0}^{1} \]

Plug in the boundary values to the equation

\[ \pi \left( 7(1) - 4(1)^{3/2} - \frac{1}{2} (1)^2 \right) - \left( 7(0) - 4(0)^{3/2} - \frac{1}{2} (0)^2 \right) \]

Simplify to get the volume

\[ \frac{5}{2} \pi \]