1. [28 points] Consider the graph of the function \( f(x) = x^3 - 6x^2 + 1 \).

\[
f'(x) = 3x^2 - 12x = 3x(x - 4) \\
f''(x) = 6x - 12 = 6(x - 2) \\
f'''(x) = 0, \text{ when } x = \frac{2}{3}
\]

(i) Find the intervals on which \( f \) is increasing.

\((-\infty, 0) \text{ and } (4, \infty)\)

(ii) Find the intervals on which \( f \) is decreasing.

\((0, 4)\)

(iii) Find the coordinates of any local maximum of \( f \).

\((0, 1)\)

(iv) Find the coordinates of any local minimum of \( f \).

\((4/3, -31)\)

(v) Find the intervals on which \( f \) is concave up.

\((2, \infty)\)

(vi) Find the intervals on which \( f \) is concave down.

\((-\infty, 2)\)

(vii) Find the coordinates of any inflection points of \( f \).

\((2/3, -15)\)
2. [16 points] Find the absolute maximum and absolute minimum values of the function \( f(x) = x^3 - 2x \) on the closed interval \([-2, 4]\). Indicate at which \( x \)-values each of these relative extrema occur.

\[ f'(x) = 3x^2 - 2 \]

Critical points: \(-1, 1, 4\)

\( f(-1) = 1 + 2 = 3 \) is a local max when \( x = -1 \)
\( f(1) = 1 - 2 = -1 \) is a local min when \( x = 1 \)
\( f(4) = 64 - 8 = 56 \) is a local max when \( x = 4 \)

3. [15 points] If each edge of a cube is increasing at a rate of 3 centimeters per second, how fast is the volume increasing when \( x \), the length of an edge is 15 centimeters long?

\[ V = x^3 \]
\[ \frac{dx}{dt} = 3 \] is the rate of cube edge increasing
\[ x = 15 \] is the current edge length

\[ \frac{dV}{dt} = \frac{d}{dt}(x^3) \] is the rate of volume increasing

\[ \Rightarrow \frac{dV}{dt} = 3x^2 \frac{dx}{dt} = 3(15)^2 \times 3 = 2025 \] is the rate of volume increasing.
4. [15 points] Evaluate the indefinite integral \( \int \left( \sqrt{x} + \frac{3}{x} \right) \, dx \).

\[
\int \left( \sqrt{x} + \frac{3}{x} \right) \, dx = \int x^{\frac{1}{2}} \, dx + 3 \int \frac{1}{x} \, dx \\
= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C_1 + 3 \ln |x| + C_2 \\
= \frac{2}{3} x^{\frac{3}{2}} + 3 \ln |x| + C
\]

5. [15 points] Use the method of substitution to evaluate the integral \( \int \frac{x + 1}{x^2 + 2x - 3} \, dx \).

\( U = x^2 + 2x - 3 \)
\( du = 2x + 2 \, dx \Rightarrow \frac{du}{2(x+1)} = dx \)

\[
\int \frac{x+1}{x^2+2x-3} \, dx = \int \frac{x+1}{U} \cdot \frac{du}{2(x+1)} = \frac{1}{2} \int \frac{1}{U} \, du \\
= \frac{1}{2} \ln |U| + C = \frac{1}{2} \ln |x^2 + 2x - 3| + C
\]
6. [15 points] Use integration by parts to evaluate the integral \( \int x^2 e^x \, dx \).

\[
\begin{align*}
& u = x^2 \\
& dv = e^x \, dx \\
& du = 2x \, dx \\
& v = e^x
\end{align*}
\]

\[
\begin{align*}
\int x^2 e^x \, dx &= x^2 e^x - \int 2xe^x \, dx \\
&= x^2 e^x - 2e^x \left[ x e^x - \int e^x \, dx \right] \\
&= x^2 e^x - 2xe^x + 2e^x + C
\end{align*}
\]

7. [16 points] Find the area enclosed by the graphs of \( f(x) = x^2 \) and \( g(x) = x + 2 \).

\[
f(x) = g(x) \implies x^2 = x + 2 \implies x^2 - x - 2 = 0
\]

\[
\implies (x-2)(x+1) = 0
\]

\[
x = 2, \, x = -1
\]

Area is \( \int_{-1}^{2} g(x) - f(x) \, dx \)

\[
\begin{align*}
&= \int_{-1}^{2} x + 2 - x^2 \, dx \\
&= \left. \left( \frac{x^3}{3} + 2x - \frac{x^3}{3} \right) \right|_{-1}^{2} \\
&= \left( \frac{8}{3} + 4 - \frac{8}{3} \right) - \left( \frac{1}{3} - 2 + \frac{1}{3} \right) \\
&= 6 - \frac{8}{3} + \frac{3}{2} - \frac{3}{2} = \frac{15}{2} - 3 = \frac{9}{2}
\end{align*}
\]