Practice Exam 1 for MATH 151
NAME: 
SECTION: 

Show all work in the space provided. Calculators are allowed. 120 points possible. (Time limit: 50 minutes) Finite or infinite limits should be written as such (using $\infty$ or $-\infty$ for infinite limits). Otherwise, if the limit is undefined, write DNE for does not exist.

1. [10 points] Solve for $x$: \( \left( \frac{1}{2} \right)^{1-x} = 4 \)

   \[
   \Rightarrow (2^{-1})^{1-x} = 2^2 \\
   \Rightarrow 2^{-1+x} = 2^2 \\
   \Rightarrow -1 + x = 2 \\
   \Rightarrow x = 3
   \]

2. [10 points] Solve for $x$: \( \log_3(2x - 1) = 4 \)

   \[
   3^4 = 2x - 1 \
   \Rightarrow 81 = 2x - 1 \
   \Rightarrow 82 = 2x \
   \Rightarrow 41 = x
   \]

3. [10 points] Write as a single logarithm: \( \frac{1}{5} \log_3(x^2 + 3) - 2 \log_3(x - 1) = \log_3 \left( \frac{(x^2 + 3)^{1/5}}{(x - 1)^2} \right) \)

4. [10 points] Approximately how long (in years) will it take to double an investment at 7% compounded continuously?

   \( \text{Recall: } A = Pe^{rt} \)

   \[
   A = 2P \quad (\text{double the investment}) \\
   P = \text{initial investment} \\
   r = 7\% = .07 \\
   t = \frac{\ln(2)}{.07} \\
   2P = Pe^{.07t} \\
   \Rightarrow \quad 2 = e^{.07t} \\
   \Rightarrow \quad \ln(2) = \ln(e^{.07t}) = .07t \\
   \Rightarrow \quad \frac{\ln(2)}{.07} = t \approx 9.9 \text{ years}
   \]
5. [10 points] \[ \lim_{{x \to \infty}} \frac{2x^3 + x^2 - 6}{5x^3 - x + 3} = \lim_{{x \to \infty}} \frac{(x + \frac{1}{x})^3 - \frac{6}{x^3}}{x^3} = \lim_{{x \to \infty}} \frac{\frac{x - 6}{x^3}}{x^3} = \lim_{{x \to \infty}} \frac{x - 6}{x^3} = \frac{2}{5} \]

6. [10 points] \[ \lim_{{x \to -2}} \frac{x^2 + x - 2}{x^2 + 5x + 6} = \lim_{{x \to -2}} \frac{(x+2)(x-1)}{(x+3)(x+2)} = \lim_{{x \to -2}} \frac{x-1}{x+3} = \frac{-3}{1} = -3 \]

7. [10 points (5 points each)] Consider the function \( f(x) = \frac{1}{x} \). The student must get part (i) correct to receive credit on part (ii).

(i). Find the average rate of change of the function \( f \) from \(-2\) to \( x \).
\[
\frac{f(x) - f(-2)}{x - (-2)} = \frac{1}{x} + \frac{1}{3} = \frac{3 + x}{3x} = \frac{3 + x}{3x} \]

(ii). Find the limit of the average rate of change of the function \( f \) from \(-2\) to \( x \) as \( x \) approaches \(-2\).
\[
\lim_{{x \to -2}} \frac{f(x) - f(-2)}{x - (-2)} = \lim_{{x \to -2}} \frac{1}{3} = \frac{-1}{4} \]

8. [10 points] \[ \lim_{{x \to 5^-}} \frac{2 - x}{5 - x} = \frac{\frac{2-5}{0^+}}{\frac{5-5}{0^+}} = -\infty \]

\[
\begin{array}{c|c|c}
4.9 & 5 - x \\
4.99 & 0.1 \\
4.999 & 0.01 \\
\downarrow & 0^+ \\
\end{array}
\]
9. [10 points (1 point each)] Let \( f(x) = \begin{cases} x + 1 & \text{if } x < 0 \\ x^2 - 2x & \text{if } 0 \leq x < 3 \\ x & \text{if } 3 \leq x \end{cases} \)

(Write DNE if the limit or value does not exist.)

(a) \( \lim_{x \to 0^{-}} f(x) = \lim_{x \to 0^{-}} (x + 1) = 1 \)

(b) \( \lim_{x \to 0^{+}} f(x) = \lim_{x \to 0^{+}} (x^2 - 2x) = 0 \)

(c) \( \lim_{x \to 0} f(x) = \text{D.N.E. since left and right limits don't agree} \)

(d) \( f(0) = \) undefined

(e) \( \lim_{x \to 3^{-}} f(x) = \lim_{x \to 3^{-}} (x^2 - 2x) = \lim_{x \to 3^{-}} (3^2 - 2(3)) = 9 - 6 = 3 \)

(f) \( \lim_{x \to 3^{+}} f(x) = \lim_{x \to 3^{+}} x = 3 \)

(g) \( \lim_{x \to 3} f(x) = 3 \)

(h) \( f(3) = 3 \)

(i) Is \( f \) continuous at 0? \( \text{No, } \lim_{x \to 0} f(x) \neq f(0) \)

(j) Is \( f \) continuous at 3? \( \text{Yes, } \lim_{x \to 3} f(x) = f(3) \)
10. [10 points] Use the definition of the derivative as \( \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \) to find \( f'(x) \), where \( f(x) = x^2 - 3x \). (Do not use the "easy rules" for taking derivatives in this problem.)

\[
f'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 3x - 3h}{h} = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 3x - 3h - x^2 + 3x}{h} = \lim_{h \to 0} \frac{2xh - 3h}{h} = \lim_{h \to 0} 2x - 3
\]

So, \( f'(x) = 2x - 3 \)
11. [10 points] The cost per day, \( C(x) \), of producing \( x \) pairs of eyeglasses is 
\[ C(x) = 0.2x^2 + 3x + 1000 \]

(a). Find the marginal cost.

\[
\begin{align*}
C'(x) &= 0.4x + 3
\end{align*}
\]

(b). Find the cost when \( x = 100 \).

\[
\begin{align*}
C(100) &= 0.2(100)^2 + 3(100) + 1000 \\
&= 2000 + 300 + 1000 \\
&= 3300
\end{align*}
\]

(c). Find the marginal cost when \( x = 100 \).

\[
C'(100) = 0.4(100) + 3 = 40 + 3 = 43
\]
12. [10 points] Find the equation of the tangent line to the graph of \( f \) at the point \((1,4)\), where \( f(x) = x^4 + 3x^2 - x + 1 \).

\[
f''(x) = 4x^3 + 6x - 1
\]

\[
f'(1) = 4(1) + 6(1) - 1 = 9 = \text{slope}
\]

Use point-slope formula.

\[
y - 4 = 9(x - 1)
\]

\[
= y = 9x - 5
\]