3 points per problem
3 points for completion

Name: Solution

Math 151 Section _____
HW 11, due at the beginning of week twelve recitation.

Answer all questions to the best of your ability. Full credit will only be given if all work is shown and organized and it is clear what your answer is. The HW is out of 12 points with part of the points from attempting the problems and part of the points from accuracy.

- Section 5.4, Problems: 1, 3, 14, 25, [redacted] 42
- Section 5.6, Problems: 1, 5, 6, 9, [redacted] 15, 16

5.4.4 \( f(x) = 4 - 2x - x^2 \) on \([-2, 2]\)

\( f''(x) = -2 - 2x = -2(x+1) \),
\( f'(x) = 0 \) when \( x = -1 \).

Critical points are: \(-2, -1, 2\)

\( f(-2) = 4 \)
\( f(-1) = 5 \Rightarrow 5 \) is the absolute max. on \([-2, 2]\)
\( f(2) = -4 \Rightarrow -4 \) is the absolute min. on \([-2, 2]\)

5.4.30

Volume = \( \pi r^2 h = 10 \Rightarrow h = \frac{10}{\pi r^2} \)

Area of top and bottom = \( 2(\pi r^2) \)
Area of walls of can = \( 2 \pi r h \)

Cost of top and bottom = \( 2(2 \pi r^2) = 4 \pi r^2 \)
Cost of walls = \( 1.5 \left(2 \pi r h\right) = 3πrh = 3\pi r \left(\frac{10}{\pi r^2}\right) \)

Total cost = \( 4πr^2 + \frac{30}{r} = C(r) \)

Minimize cost: \( C'(r) = 8\pi r^3 - \frac{30}{r^2} = 8\pi r^3 - \frac{30}{r^2} = 0 \Rightarrow r = \sqrt[3]{\frac{30}{8\pi}} \)

\( \Rightarrow h = \frac{10}{\pi \left(\frac{30}{8\pi}\right)^{\frac{1}{3}}} \)
\[ \frac{\text{change in radius}}{\text{change in time}} = \frac{dr}{dt} = \frac{1 \text{ cm}}{\text{sec}} \quad r = 6 \]

\[ \frac{\text{change in volume}}{\text{change in time}} = \frac{dV}{dt} \]

\[ V = \frac{4}{3} \pi r^3 \]

\[ \Rightarrow \quad \frac{dV}{dt} = 4 \pi r^2 \frac{dr}{dt} = (4 \pi \cdot 36 \cdot 1) \frac{\text{cm}^3}{\text{sec}} = 144\pi \frac{\text{cm}^3}{\text{sec}}. \]