1. [8 points] Solve for $x$: \( \left( \frac{1}{2} \right)^{1-x} = 4 \)

2. [8 points] Solve for $x$: \( \log_3(3x - 2) = 2 \)

3. [8 points] Write as a single logarithm: \( 3 \log_5(3x + 1) - 2 \log_5(2x - 1) - \log_5 x \)

4. [8 points] What annual rate of interest compounded continuously is required to triple an investment in 20 years?

5. [8 points] \( \lim_{x \to 3} \frac{x^2 - 9}{x^2 - 3x} = \)
6. [10 points (1 point each)] Let \( f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ x & \text{if } 1 < x \leq 2 \\ x - 1 & \text{if } 2 < x \end{cases} \)

(Write DNE if the limit or value does not exist.)

(a). \( \lim_{x \to 1^-} f(x) = \)

(b). \( \lim_{x \to 1^+} f(x) = \)

(c). \( \lim_{x \to 1} f(x) = \)

(d). \( f(1) = \)

(e). \( \lim_{x \to 2^-} f(x) = \)

(f). \( \lim_{x \to 2^+} f(x) = \)

(g). \( \lim_{x \to 2} f(x) = \)

(h). \( f(2) = \)

(i). Is \( f \) continous at 1?

(j). Is \( f \) continous at 2?
7. [12 points] For a certain production facility, the cost function is

\[ C(x) = 2x + 5 \]

and the revenue function is

\[ R(x) = 8x - x^2 \]

where \( x \) is the number of units produced and sold, and \( R \) and \( C \) are measured in millions of dollars. Find the following:

(a). Find the marginal revenue.

(b). Find the marginal cost.

(c). Find the break-even point(s) [the number(s) \( x \) for which \( R(x) = C(x) \)]

(d). Find the number \( x \) for which marginal revenue equals the marginal cost.
8. [8 points] Find the equation of the tangent line to the graph of \( f \) at the point \((1, 0)\), where \( f(x) = x \ln x \).

9. [8 points] Find \( f'(x) \) where \( f(x) = \frac{e^x}{x+1} \).

10. [10 points] Find \( \frac{dy}{dx} \) where \( y = 2x^3(4x - 3)^5 \).
11. [8 points] If a rock falls from a height of 40 meters on the planet Jupiter, then its height $H$ after $t$ seconds is approximately

$$H(t) = 40 - 10t^2$$

(a). What is the average velocity of the rock from $t = 0$ to $t = 1$?

(b). What is the instantaneous velocity at time $t = 1$?

(c). What is the acceleration of the rock?

(d). When does the rock hit the ground?
12. [14 points] Consider the graph of the function \( f(x) = 2x^3 - 6x \).

(i). Find the intervals on which \( f \) is increasing.

(ii). Find the intervals on which \( f \) is decreasing.

(iii). Find the coordinates of any local maximums of \( f \).

(iv). Find the coordinates of any local minimums of \( f \).

(v). Find the intervals on which \( f \) is concave up.

(vi). Find the intervals on which \( f \) is concave down.

(vii) Find the coordinates of any inflection points of \( f \).
13. [10 points] What is the slope of the tangent line to the curve \( xy^2 - x^2 y = 2 \) at the point \((1, 2)\)?

14. [10 points] Find the absolute maximum and absolute minimum values of the function \( f(x) = x^2 - 2x \) on the closed interval \([-4, 4]\). Indicate at which \( x \)-values each of these relative extrema occur.
15. [10 points] If each edge of a cube is increasing at a rate of 3 centimeters per second, how fast is the volume increasing when $x$, the length of an edge, is 15 centimeters long?

16. [10 points] Evaluate the indefinite integral $\int \frac{dx}{2x - 5}$. 
17. [10 points] Evaluate the infinite integral \( \int x \sqrt{x^2 + 1} \, dx \).

18. [10 points] Evaluate the infinite integral \( \int x \ln x \, dx \).
19. [10 points] Find the area enclosed by the graph of \( f(x) = x^2 - 1 \) and the \( x \)-axis.