1. (10 points) Find the derivative of \( f(x) = x^2 \ln(x) \).
   Use the product rule: \( f'(x) = x^2 \cdot \frac{1}{x} + \ln(x)(2x) \).

2. (10 points) Find the derivative of \( f(x) = \frac{e^x + x}{x^2 + 1} \).
   Use the quotient rule: \( f'(x) = \frac{(x^2 + 1)(e^x + 1) - (e^x + x)(2x)}{(x^2 + 1)^2} \).

3. (10 points) Find the derivative of \( f(x) = (x^3 + 5x^2 + 2)^{-1/2} \).
   Use the power rule: \( f'(x) = (-1/2)(x^3 + 5x^2 + 2)^{-3/2}(3x^2 + 10x) \).

4. (10 points) Find the derivative of \( f(x) = \ln[(3x^2 + x)^4] \).
   Use the chain rule for the natural log function: \( f'(x) = \frac{1}{(3x^2 + x)^4} \cdot 4(3x^2 + x)^3(6x + 1) \).

5. (10 points) Find the derivative of \( e^{(\ln(x)+e^x+x^2)} \).
   Use the chain rule for the exponential function: \( f'(x) = e^{(\ln(x)+e^x+x^2)} \cdot (\frac{1}{x} + e^x + 2x) \).

6. (10 points) What is the 4th derivative of \( e^{2x} \)?
   \( f'(x) = 2e^{2x} \Rightarrow f''(x) = 4e^{2x} \Rightarrow f'''(x) = 8e^{2x} \Rightarrow f^{(4)}(x) = 16e^{2x} \).
7. (10 points) I throw a ball into the air and the height, in feet, of the ball at time \( t \), in seconds, is given by \( h(t) = -16t^2 + 12t + 10 \).

(a) What is the velocity, \( v(t) \), of the ball? \( v(t) = -32t + 12 \).

(b) What is the acceleration, \( a(t) \), of the ball? \( a(t) = -32 \).

(c) What is the velocity of the ball at time \( t = 2 \)? \( v(2) = -52 \text{ ft/sec} \)

(d) At what time is the velocity 0? \( v(t) = 0 \Rightarrow -32t + 12 = 0 \Rightarrow t = 3/8 \text{ seconds} \).

8. (10 points) Given \( f(x) = x^3 - 3x \), find where the graph of the function is increasing, decreasing and find any local maximums or minimums.

\[ f'(x) = 3x^2 - 3 = 3(x^2 - 1) = 3(x + 1)(x - 1). \]

The splitting points are \( x = 1 \) and \( x = -1 \). 3 is always positive, \( x + 1 \) is negative on \((-\infty, -1)\) and positive on \((-1, \infty)\) and \( x - 1 \) is negative on \((-\infty, 1)\) and positive on \((1, \infty)\). Thus the overall derivative is positive on \((-\infty, -1)\) and \((1, \infty)\) (so \( f(x) \) is increasing) and negative on \((-1, 1)\) (so \( f(x) \) is decreasing). This further implies a local maximum at \((-1, f(-1))\) and a local minimum at \((1, f(1))\).

9. (10 points) Find the horizontal and vertical tangent lines of \( f(x) = \frac{x^2 + 4}{x} \).

We want to know when the derivative is 0 for horizontal tangent lines and unbounded (\( \infty \) or \( -\infty \)) for vertical tangent lines. \( f'(x) = \frac{x(2x) - (x^2 + 4)(1)}{x^2} = \frac{(x + 2)(x - 2)}{x^2} \). This means we have horizontal tangent line at \((2, 4)\) and \((-2, -4)\). The derivative "blows up" when \( x = 0 \) which gives a vertical tangent line, but 0 is not in the domain so we do not have to consider it thus we have no vertical tangent lines.
10. (10 points) Find \( \frac{dy}{dx} \) using implicit differentiation where \( xy + 3y^2 = 6x \). What is the slope of the tangent line at (3, 2)? What is the equation of the tangent line at (3, 2)?

Differentiating both sides with respect to \( x \) (note that I am not making the \( y = f(x) \) substitution) we get \( xy' + y + 6yy' = 6 \). Solving for \( y' \) gives \( y' = \frac{6 - y}{x + 6y} \). The slope of the tangent line at (2, 3) is \( \frac{6 - 2}{3 + 6(2)} = \frac{4}{15} \), thus the equation of the tangent line is \( y - 2 = \frac{4}{15}(x - 3) \).