1. Section 5.3; Page 378; Problems: 14, 25, 30, 47, 70

2. Section 5.4; Page 391; Problems: 1, 12, 14, 15, 25, 30, 42

3. Section 5.6; Page 404; Problems: 1, 5, 6, 7, 9, 10, 15, 16

Problem 5.3.30 The function is \( f(x) = x^3 + 6x^2 + 2 \).

The domain is all reals. The \(-x\)-ints are too hard to find, the \(-y\)-intercept is \((0, f(0)) = (0,2)\).

\( f'(x) = 3x^2 + 12x = 3x(x+4) \) thus the split points are 0 and -4 so \( f \) is increasing on \((-\infty, -4)\) and \((0, \infty)\) and decreasing on \((-4, 0)\). Local maximum at \((-4, \frac{10}{3})\) and local minimum at \((0,2)\).

\( f''(x) = 6x + 12 = 6(x+2) \) so we have one split point at -2 thus \( f \) is concave down on \((-\infty, -2)\) and concave up on \((-2, \infty)\) so we have an inflection point at \((-2, -20)\). There are no asymptotes and end behavior is like \( x^3 \), down to the left and up to the right. Hopefully the graph looks ok.

Problem 5.4.25 If we label the corner piece cut out as an \( x \) by \( x \) square the dimensions of our cardboard box are \( (12-2x) \) by \( (12-2x) \) by \( x \), thus the volume of our box is \( V(x) = x(12-2x)^2 \).

\( V'(x) = 2(12-2x)(-2) + (12-2x)^2(1) = (-12)(x-2)(6-x) \) where \( 0 \leq x \leq 6 \), thus our critical points are 0, 2 and 6 but 0 and 6 give 0 volume and \( V(2) = 128 \) cubic cm. which is a maximum since \( V''(x) > 0 \) on our interval of concern. Thus the dimensions are 8 x 8 x 2.

Problem 5.4.30 \( V = \pi r^2 h \) and \( V = 10 \) so \( 10 = \pi r^2 h \Rightarrow h = 10/(\pi r^2) \). The cost of the top and the bottom is \( (2)(\pi r^2)(82) = 4\pi r^2 \) and the cost of the sides is \( (2\pi rh)(1.5) = 3\pi rh \) so the total cost is \( C = 4\pi r^2 + 3\pi rh \) and substituting our \( h \) and simplifying we have \( C(r) = 4\pi r^2 + 30r^{-1} \) with \( r > 0 \). We find the first and second derivatives to find local extrema and to tell whether or not it is a maximum or minimum. \( C'(r) = 8\pi r - 30r^{-2} = \frac{2(4\pi r^3 - 15)}{r^2} \) thus we have one critical point when \( r = \sqrt{\frac{15}{4\pi}} \). But \( C''(r) = 8\pi + 60r^{-3} > 0 \) when \( r > 0 \) so we have found an \( r \) which minimizes cost and the corresponding \( h \) is \( \frac{10}{\pi (\frac{15}{4\pi})^{2/3}} \).

Problem 5.6.9 Let the edges of the cube be labeled by \( x \), then the volume of the cube is \( V = x^3 \). We also know \( \frac{dx}{dt} = 3 \) and want to find \( \frac{dV}{dt} \) when \( x = 10 \). Differentiating \( V = x^3 \) with respect to \( t \) we get \( \frac{dV}{dt} = 3x^2 \frac{dx}{dt} \) thus \( \frac{dV}{dt} = 3(10^2)(3) = 900 \) cm\(^3\) per second.

Problem 5.6.10 Let \( r \) be the radius of the sphere and \( V \) be the volume, then \( V = \frac{4}{3}\pi r^3 \). We are given that \( \frac{dr}{dt} = 1 \) and want to find \( \frac{dV}{dt} \) when \( r = 6 \). Differentiating both sides with respect to \( t \) gives \( \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \) and substituting gives \( \frac{dV}{dt} = 144\pi \).