Riemann Sums and Limits

Name:
Math 165 quiz
Date: 11/14/2013
Section:

Instructions: This quiz is out of 10 points and you have 15 minutes to work on it. There is a second page! No calculators are allowed. Show all work; partial credit may be given but only if correct work and reasoning are shown and explained. Please write legibly and organize your work well!

Hints: \[
\sum_{k=1}^{n} k = \frac{n(n + 1)}{2}, \\
\sum_{k=1}^{n} k^2 = \frac{n(n + 1)(2n + 1)}{6}, \\
\sum_{k=1}^{n} k^3 = \left(\frac{n(n + 1)}{2}\right)^2
\]

1. (5 points) Use the Midpoint Rule to estimate the area under the graph of \( f(x) = \frac{35}{x+1} \) and above the graph of \( g(x) = 0 \) from \( x_0 = 0 \) to \( x_n = 1 \) using two rectangles of equal width.

Since we have two rectangles of equal width our intervals are \((0, 1/2)\) and \((1/2, 1)\) and they have midpoints 1/4 and 3/4 respectively. Note that the base of each rectangle has width 1/2.

The Midpoint Rule tells us that to find the height of the first rectangle we need to plug 1/4 into our function. So the height of our first rectangle is

\[
f(1/4) = \frac{35}{1/4 + 1} = \frac{35}{5/4} = \frac{35 \cdot 4}{5} = 28.
\]

Multiplying this by the width of 1/2 we find the area of the first rectangle to be \( 28 \cdot 1/2 = 14 \).

We do something similar for the second rectangle.

\[
f(3/4) = \frac{35}{3/4 + 1} = \frac{35}{7/4} = \frac{35 \cdot 4}{7} = 20.
\]

Thus the area of the second rectangle is \( 20 \cdot 1/2 = 10 \).

Therefore our total estimate is \( 14 + 10 = 24 \) square units.
2. (5 points) Consider the following statement about Riemann sums written by Bernhard.

*For the function* \( f(x) = x^2 \), the Riemann sum obtained by dividing the interval \([0, 5]\) into \( n \) equal subintervals and using the right-hand endpoint for each \( c_k \) is given by

\[
\sum_{k=1}^{n} \left( \frac{5}{n} \right)^3 \frac{4}{n}
\]

(a) Find the mistakes in Bernhard’s formula and write the correct formula.

The correct formula is

\[
\sum_{k=1}^{n} \left( \frac{5}{n} \right)^2 \frac{5}{n}.
\]

(b) Assuming you found the correct formula in part (a), find the limit of this sum as \( n \to \infty \).

Using the properties of summation notation and limits we have

\[
\lim_{n \to \infty} \sum_{k=1}^{n} \left( \frac{5}{n} \right)^2 \frac{5}{n} =
\]

\[
\lim_{n \to \infty} \sum_{k=1}^{n} \frac{25}{n^2} \cdot \frac{5}{n} =
\]

\[
\lim_{n \to \infty} \frac{125}{n^3} \sum_{k=1}^{n} k^2 =
\]

\[
\lim_{n \to \infty} \frac{125}{n^3} \cdot \frac{n(n + 1)(2n + 1)}{6} =
\]

\[
\lim_{n \to \infty} \frac{125}{n^3} \cdot \frac{2n^3 + 3n^2 + n}{6} =
\]

\[
\lim_{n \to \infty} \frac{250n^3 + 375n^2 + 125n}{6n^3} = \frac{250}{6} = \frac{125}{3}.
\]