Optimization and Concavity

Name: 
Math 165 quiz
Date: 10/31/2013
Section: ______

Instructions: This quiz is out of 10 points and you have 15 minutes to work on it. There is a second page! No calculators are allowed. Show all work; partial credit may be given but only if correct work and reasoning are shown and explained. Please write legibly and organize your work well!

1. (5 points) Your dream of becoming a Compsognathus (dinosaur) breeder has finally come true. You are constructing a set of rectangular pens in which to breed your vicious friends. The overall area you are working with is 120 square feet, and you want to divide the area up into six pens of equal size as shown below.

The cost of the outside fencing is $10 a foot. The inside fencing costs $5 a foot. Find the dimensions of the pen as to minimize the cost of the fencing.

\[ A = lw = 120 \] where \( w \) is the length of the dividers.

The cost equation is \( 10(2l + 2w) + 5(5w) \).

Substituting \( l = 120/w \) and simplifying we come up with \( C(w) = 2400w^{-1} + 45w \).

Now we take the derivative of the cost function and set it equal to zero to find critical points. \( C'(w) = 45 - 2400/w^2 = 0 \Rightarrow w = \sqrt{2400/45} \Rightarrow l = 120/\sqrt{2400/45} \).
2. (5 points) Let \( f(x) \) be a function that is increasing on \((-\infty, -2)\), decreasing on \((-2, 3)\) and increasing again on \((3, \infty)\). What is the derivative of \( f(x) \)? Based on the derivative of \( f(x) \) where is the function concave down and concave up?

Since the function is increasing on \((-\infty, -2)\) and \((3, \infty)\) then \( f'(x) > 0 \) on the same intervals.

Since the function is decreasing on \((-2, 3)\) we know \( f'(x) < 0 \) on the same interval.

Together this means the derivative is zero at \( x = -2 \) and at \( x = 3 \), thus \( f'(x) \) must have the factors \((x + 2)\) and \((x - 3)\), we will initially guess that \( f'(x) = (x + 2)(x - 3) \).

Note that we still need to make sure there is not a negative factor in our derivative e.g. maybe \( f'(x) = -(x+2)(x-3) \). To check we use test points in our intervals and plug them into our initial guess.

It turns out we were right, so \( f'(x) = (x + 2)(x - 3) = x^2 - x - 6 \).

To figure out concavity we need to find when the second derivative is positive and negative by setting it equal to zero and solving for \( x \) to find potential inflection points.

\[
    f''(x) = 2x - 1 = 0 \Rightarrow x = 1/2.
\]

Using test points in the intervals \((-\infty, 1/2)\) and \((1/2, \infty)\) tells us that \( f''(x) < 0 \) on \((-\infty, 1/2)\) and \( f''(x) > 0 \) on \((1/2, \infty)\).

This means our function is concave down on \((-\infty, 1/2)\) and concave up on \((1/2, \infty)\).