Exam Review

Name:__________________________
Math 165
Date: 12/5/2013
Section:_______

This is a worksheet that will help you review for the final exam.

1. Find the linearization of \( g(x) = x^3 + 3x - 2 \) at the point \( x = 1 \) to approximate \( g(1.1) \).

2. Find the equation of the tangent line of \( g(x) = x^3 + 3x - 2 \) at the point \( x = 1 \). Plug 1.1 into the equation of the tangent line that you found.

3. What is the relationship between finding the linearization of a function at a point and finding the equation of a tangent line at a point?
4. What are the two fundamental theorems of calculus?

5. What is \( \frac{d}{dx} \left[ \int_a^x f(t) \, dt \right] \)? How can you make this more challenging?

6. If \( F'(x) = f(x) \), \( F(0) = 1 \), \( F(1) = 2 \) and \( F(2) = 4 \) what is \( \int_0^2 f(x) \, dx \)?

7. What can we say about \( f(x) \) and \( g(x) \) if \( \lim_{x \to \infty} \frac{f(x)}{g(x)} = \infty \)?
8. What can we say about \( f(x) \) and \( g(x) \) if \( \lim_\limits_{x \to \infty} \frac{f(x)}{g(x)} = 0 \)?

9. What can we say about \( f(x) \) and \( g(x) \) if \( \lim_\limits_{x \to \infty} \frac{f(x)}{g(x)} = c \) for some constant \( c \)?

10. Make up and solve a related rates problem involving triangles.

11. Make up and solve a related rates problem involving circles.
12. What is the first derivative test used for?

13. What is the second derivative test used for?

14. Sketch the graph of the function where $f'(x) > 0$ and $f''(x) > 0$ for all $x$. Write down in words what the first and second derivative tell us about the function.

15. Sketch the graph of the function where $f'(x) < 0$ and $f''(x) < 0$ for all $x$. Write down in words what the first and second derivative tell us about the function.

16. Sketch the graph of the function where $f'(x) < 0$ and $f''(x) > 0$ for all $x$. Write down in words what the first and second derivative tell us about the function.

17. Sketch the graph of the function where $f'(x) > 0$ and $f''(x) < 0$ for all $x$. Write down in words what the first and second derivative tell us about the function.
18. What is the derivative of $3x^2 + x + 1$?

19. If $a > 1$ what is $\lim_{x \to \infty} a^x$? Why?

20. If $a = 1$ what is $\lim_{x \to \infty} a^x$? Why?

21. If $0 < a < 1$ what is $\lim_{x \to \infty} a^x$? Why?

22. When do you use L’hopital’s rule?

23. Calculate $\lim_{x \to 0^+} x \ln(x)$. 
24. Set up the integrals to find the area trapped between the graphs of the functions \( \cos(x) \) and \( \sin(x) \) from 0 to \( 2\pi \).

25. Evaluate the integrals from the previous problem.

26. What are the steps to solving a separable differential equation with initial conditions?

27. Find the function \( y \) that satisfies \( \frac{dy}{dx} = 6y^2x \) and \( y(1) = \frac{1}{25} \).
28. Suppose that electricity is draining from a capacitor at a rate that is proportional to the voltage $V$ across its terminals and that, if $t$ is measured in seconds,

$$\frac{dV}{dt} = -\frac{1}{40} V.$$ 

Solve this equation for $V$, using $V_0$ to denote the value of $V$ when $t = 0$. How long will it take the voltage to drop to 10% of its original value?

29. \[\int \frac{dx}{e^x + e^{-x}}.\] If you don’t know what the integral is right away, you can try two things: simplify or try $u$-substitution.

30. \[\int \frac{x}{(x - 4)^3} \, dx.\] A good exercise might be to list all the possible $u$-substitutions, even those that don’t work! Explain why a certain choice is good or bad.
31. What is wrong with the following steps:

I choose $u = g(x)$ and so $du = g'(x)$. Then

$$\int_a^b \left( g(x) \right)^2 \cdot g'(x) dx = \int_a^b u^2 du$$

$$= \frac{1}{3} u^3 \bigg|_a^b$$

$$= \frac{1}{3} g(x)^3 \bigg|_a^b$$

$$= \frac{1}{3} g(b)^3 - \frac{1}{3} g(a)^3$$

Did we find the correct answer? If you say there is nothing wrong, then are you really sure?