Let \( f(x) \) be a function and \( c \) be a positive number, then:

1. \( f(x) + c \) shifts the graph of \( f(x) \) **vertically up** by \( c \) units, e.g. if the point \((x, y)\) is on the graph of \( f(x) \) then \((x, y + c)\) is on the graph of \( f(x) + c \).
2. \( f(x) - c \) shifts the graph of \( f(x) \) **vertically down** by \( c \) units, e.g. if the point \((x, y)\) is on the graph of \( f(x) \) then \((x, y - c)\) is on the graph of \( f(x) - c \).
3. \( f(x + c) \) shifts the graph of \( f(x) \) **horizontally to the left** by \( c \) units, e.g. if the point \((x, y)\) is on the graph of \( f(x) \) then \((x - c, y)\) is on the graph of \( f(x + c) \).
4. \( f(x - c) \) shifts the graph of \( f(x) \) **horizontally to the right** by \( c \) units, e.g. if the point \((x, y)\) is on the graph of \( f(x) \) then \((x + c, y)\) is on the graph of \( f(x - c) \).
5. If \( c > 1 \) then \( cf(x) \) **stretches the graph vertically** by a scale of \( c \) units e.g. if \((x, y)\) is on the graph of \( f(x) \) then \((x, cy)\) is on the graph of \( cf(x) \).
6. If \( 0 < c < 1 \) then \( cf(x) \) **compresses the graph vertically** by a scale of \( c \) units e.g. if \((x, y)\) is on the graph of \( f(x) \) then \((x, cy)\) is on the graph of \( cf(x) \).
7. If \( c > 1 \) then \( f(cx) \) **compresses the graph horizontally** by a scale of \( c \) units e.g. if \((x, y)\) is on the graph of \( f(x) \) then \((\frac{x}{c}, y)\) is on the graph of \( f(cx) \). Since \( c > 1 \) notice that \( xc \) is smaller than \( x \). You can think of it as getting to each \( y \) value faster.
8. If \( 0 < c < 1 \) then \( f(cx) \) **stretches the graph horizontally** by a scale of \( c \) units e.g. if \((x, y)\) is on the graph of \( f(x) \) then \((\frac{x}{c}, y)\) is on the graph of \( f(cx) \). Since \( 0 < c < 1 \) notice that \( xc \) is smaller than \( x \). You can think of it as getting to each \( y \) value slower.
9. The graph of \(-f(x)\) **reflects the graph of \( f(x) \) over the \( x \)-axis** e.g. if \((x, y)\) is on the graph of \( f(x) \) then \((x, -y)\) is on the graph of \(-f(x)\).
10. The graph of \(f(-x)\) **reflects the graph of \( f(x) \) over the \( y \)-axis** e.g. if \((x, y)\) is on the graph of \( f(x) \) then \((-x, y)\) is on the graph of \(f(-x)\).
11. For consistency, and to avoid some pitfalls, a good order to do your transformations in is:

   1. Reflections
   2. Stretches and Compressions
   3. Shifts