Name: ________________________________

Math 140 Section C, Final Review, Due Saturday, December 9th

Covers sections 2.5, 3.1, 3.2, 3.3, 3.4, 3.5, 6.1, 6.2, 4.3, 4.4, 5.1, 5.2, 5.3, 5.4, 5.5, 5.6, 6.3, 6.4, 6.5, 6.6,
(and minimally) 13.1, 13.2, 13.3, 13.5

Answer all questions to the best of your ability. Full credit will only be given if all work is shown and
organized and it is clear what your answer is. The review is out of 25 points.

1. The kinetic energy of a moving object varies jointly with its mass $m$ and the square of its velocity $v$.
If an object weighing 20 kilograms and moving with a velocity of 5 meters per second has a
kinetic energy of 1000 joules, find its kinetic energy when the velocity is 15 meters per second.

$$E = \frac{1}{2}mv^2$$

$m = 20$ kg, $v = 5$ m/s

$$E = 1000$$ J

when $v = 15$ we have

$$E = \frac{1}{2} \cdot 20 \cdot (15)^2$$

$$= 40 \cdot 225$$

$$= 9000$$ J

2. The cube of $z$ varies inversely with the sum of the squares of $x$ and $y$. Find the general equation
if $z = 2$ when $x = 9$ and $y = 4$.

$$z^3 = \frac{k}{x^2 + y^2}$$

$k = \frac{z^3}{x^2 + y^2}$

$k = \frac{2^3}{9^2 + 4^2}$

$k = \frac{8}{97}$

$$z^3 = \frac{1}{(\frac{8}{97})}(x^2 + y^2) = \frac{97}{8}(x^2 + y^2)$$

3. Answer the following questions about the relation \{(2, 1), (3, 1), (4, 1), (5, 2), (6, 4)\}.

(a) Is the relation a function? Why or why not? Yes, each input has at most one output.

(b) What is the domain of the relation? $\{2, 3, 4, 5, 6\}$

(c) What is the range of the relation? $\{1, 2, 4\}$
4. Find the domains of the following functions:

   (a) \( f(x) = \ln(x - 3) \)
   (b) \( g(x) = 2^{x-1} \)
   (c) \( h(x) = \frac{x + 2}{x^2 - 4} \)
   (d) \( F(x) = \sqrt{(x+1)(x-2)} \)

   \[ a. \quad x - 3 \geq 0 \Rightarrow x \geq 3 \quad \text{so} \quad \text{dom}(f) = \mathbb{R} \backslash (-\infty, 3] \]
   (b) all reals
   (c) \( x^2 - 4 \neq 0 \Rightarrow (x+2)(x-2) \neq 0 \Rightarrow \text{dom}(h) = \mathbb{R} \backslash \{ -2, 2 \} \)
   (d) \((x+1)(x-2) \geq 0 \)

5. What is the domain of the function \( \frac{x^4 + 1}{x^2 - 1} \)?

   \[ x^4 + 1 \geq 0 \quad \text{is always true} \]
   \[ x^2 - 1 \neq 0 \quad \Rightarrow \quad x^2 - 1 = (x+1)(x-1) \neq 0 \quad \Rightarrow \quad x \neq 1, x \neq -1 \]
   
   so, the domain is \( \mathbb{R} \backslash \{ 1, -1 \} \)

6. If \( f(x) = x^2 - 2 \) and \( g(x) = 3x \) what are (simplify):

   (a) \( (f + g)(x) = x^2 - 2 + 3x = x^2 + 3x - 2 \)
   (b) \( (f - g)(x) = x^2 - 2 - 3x = x^2 - 3x - 2 \)
   (c) \( (f \cdot g)(x) = (x^2 - 2)(3x) = 3x^3 - 6x \)
   (d) \( \left( \frac{f}{g} \right)(x) = \frac{x^2 - 2}{3x} \quad \text{with domain} \quad \mathbb{R} \backslash \{ 0 \} \]
   (e) \( (f + g)(1) = 1 + 3 - 2 = 2 \)
   (f) \( (f - g)(0) = -2 - 0 = -2 \)
   (g) \( (f \cdot g)(-2) = -2 \cdot 6 - 12 = 12 \)
   (h) \( \left( \frac{f}{g} \right)(-1) = \frac{1 + 2}{(-3)} = \frac{-1}{-3} = \frac{1}{3} \)
7. Are the following functions even, odd or neither, be sure to justify your answer

(a) \( f(x) = x^3 - 4x \) 
\[ f(-x) = -x^3 + 4x \neq f(x) \] \( \Rightarrow \) odd

(b) \( g(x) = \frac{4 + x^2}{1 + x^4} \) 
\[ g(-x) = \frac{4 + (-x)^2}{1 + (-x)^4} = \frac{4 + x^2}{1 + x^4} = g(x) \] \( \Rightarrow \) even

(c) \( H(x) = 1 + x + x^2 \) 
\[ H(-x) = 1 - x + x^2 \neq H(x) \] \( \Rightarrow \) neither

8. Solve the following inequalities:

(a) \( x^2 - x - 6 < 0 \)

(b) \( \frac{(x + 2)(x - 2)}{x - 3} \geq 0 \)

(c) \( \frac{1 - x}{2 + x} \leq 0 \)

\( (x-3)(x+2) \leq 0 \)
\[ \begin{array}{ccc} x+2 & 0 & + \\ x-3 & - & 0 \\ (x-3)(x+2) & + & 0 \end{array} \]
\[ -2 \leq x < 3 \] \( \Rightarrow \) solution is

\( \frac{(x+3)(x-3)}{x-3} \geq 0 \)
\[ \begin{array}{ccc} x+3 & 0 & + \\ x-3 & - & 0 \\ x-3 & - & 0 \\ (x-3)(x+3) & + & 0 \end{array} \]
\[ -3 \leq x < 2, \quad \text{or} \quad x > 3 \] \( \Rightarrow \) solution is

\( \frac{1-x}{x+2} \leq 0 \)
\[ \begin{array}{ccc} x+2 & 0 & + \\ 1-x & + & 0 \\ 1-x & + & 0 \end{array} \]
\[ x < -2, \quad x \geq 1 \] \( \Rightarrow \) solution is
9. Graph each of the following functions using translations and reflections. Identify at least 2 points on the final graph. State the domain and, based on the graph, find the range. Be sure to show a graph of your original function and how the graph changes at each step.

(a) \( f(x) = |x| + 4 \)

\[ \begin{align*}
\text{domain} & \quad \text{is all reals} \\
\text{range} & \quad \text{is} \quad \mathbb{R} \cup \{ 4, \infty \}
\end{align*} \]

(b) \( f(x) = -|x| + 4 \)

\[ \begin{align*}
\text{domain} & \quad \text{is all reals} \\
\text{range} & \quad \text{is} \quad (-\infty, 4]
\end{align*} \]
10. Consider the function \( f(x) = \sqrt{-x + 1} - 2 \).

(a) If we were going to sketch this function what function would we start out with?
(b) Which axis does the function get reflected across?
(c) Is the function shifted left or right? If so how many units?
(d) Does the function get shifted up or down? If so how many units?

\[ \sqrt{x} \]

\[ y - \text{axis} \]

\[ \text{left one unit} \]

\[ \text{down two units} \]

\[ W = kt^2 \]

\[ W = 0 \]

\[ t = 5 \]

\[ 10 = k \cdot 5^2 = 25k \]

\[ k = \frac{2}{5} \]

\[ W = \frac{2}{5} t^2 \]

11. Find the general formula to describe the variation \( W \) varies directly with \( t^2 \) and we know \( W = 10 \) when \( t = 5 \).

\[ W = \frac{2}{5} t^2 \]

12. Solve the following for \( x \):

(a) \( \log(x + 2) = \frac{16}{10} - \log(x + 3) \)

(b) \( 9^{2x} = 27^{x - 1} \)

(c) \( 4^{\log_{16}(25) + \log_{4}(7)} \)

(d) \( \log_{16}(x + 2) + \log_{16}(x + 3) = 1 \)

(e) \( x^2 + 5x + 6 = 0 \)

(f) \( x^2 + 5x - 4 = 0 \)

\[ x = -3 \]

\[ x = -5 \pm \sqrt{41} \]

\[ x = \frac{7}{3} \]
(c) \( f(x) = -\sqrt{x + 3} \)

** Domain: \([-3, \infty)\)**

** Range: \((-\infty, 0]\)**

(d) \( g(x) = -(x - 1)^3 - 1 \) (Hint: \( g(x) = -[(x - 1)^3 + 1] \))

** Domain: All Reals**

** Range: All Reals**
13. Find a rational function whose only zeros are 2 and 3, is not defined at 5 and has a horizontal asymptote at \( y = 2 \).

\[
R(x) = \frac{a(x-2)(x-3)}{(x-5)^2}
\]

14. Find a rational function with a vertical asymptote at \( x = 1 \), with one zero at \( x = 4 \) and a horizontal asymptote at \( y = -2 \).

\[
R(x) = \frac{-2(x-4)}{x-1}
\]

15. Find a polynomial of degree 5 with real coefficients and zeros \( 1, 1-i \) and \( 2i \).

\[
f(x) = (x-1)(x-(1-i))(x-(1+i))(x-i)(x-2i)
\]

16. Write the following as a single logarithm and simplify: \( \log_3(5) + \log_3(6) - \log_3(10) \)

\[
= \log_3 \left( \frac{5 \cdot 6}{10} \right) = \log_3 \left( \frac{30}{10} \right) = \log_3(3) = 1
\]
17. Solve for \( x \): \( \log \sqrt{x+3} = \frac{1}{2} \Rightarrow \sqrt{x+3} = 10^{\frac{1}{2}} = 10 \Rightarrow x + 3 = 10 \Rightarrow x = 7 \)

18. Expand \( \frac{2x^{1/2}(x+1)^2}{(x-2)(x-3)} \) into a sum and difference of logarithms, express powers as factors.

\[
= \ln(x^{1/2}) + \ln((x+1)^2) - \left[ \ln(x-3) + \ln(x-2) \right]
\]

\[
= \ln(x^{1/2}) + \ln(x+1) + \ln(x+1) - \ln(x-3) - \ln(x-2)
\]

19. Find the inverse of the function \( f(x) = \frac{2x-1}{x} \). Check to make sure that \( f(f^{-1}(x)) = x \).

\( y = \frac{2x-1}{x} \) switch \( x \) and \( y \)

\( x = \frac{2y-1}{y} \Rightarrow x \gamma = 2\gamma - 1 \)

\( xy - \gamma y = -1 \Rightarrow (x^{-1})y = -1 \Rightarrow x = \frac{-1}{x-2} = f^{-1}(x) \)

\( f(f^{-1}(x)) = \frac{x - \frac{1}{x-2}}{x-2} = 1 - \frac{1}{x-2} = \frac{-2}{x-3} - \frac{x-3}{x-2} \)

\( x = \frac{-2}{x-3} - \frac{x-3}{x-2} \)

\( -\frac{1}{x-2} = \frac{x-3}{x-2} \cdot \frac{-1}{-1} = \frac{-x}{x-3} \cdot \frac{-1}{-1} = x \)
20. Find the inverse of the function \( g(x) = \frac{3x+4}{1-x} \) and be sure to check that it is the inverse.

\[
\begin{align*}
y &= \frac{3x+4}{1-x} \quad \text{switch } x \text{ and } y \quad \Rightarrow \quad x &= \frac{3y+4}{1-y} \\
&\Rightarrow \quad x - xy = 3y + 4 \\
&\Rightarrow \quad -xy - 3y = 4 - x \\
&\Rightarrow \quad (x-3)y = 4 - x \\
&\Rightarrow \quad y = \frac{4-x}{x-3} = \frac{x-4}{x+3},
\end{align*}
\]

\[
f(f^{-1}(x)) = f\left(\frac{x-4}{x+3}\right) = \frac{\frac{3(x-4)}{x+3} + 4}{1 - \frac{x-4}{x+3}} = \frac{\frac{3x-12}{x+3} + \frac{4(x+3)}{x+3}}{\frac{x+3}{x+3} - \frac{x-4}{x+3}}
\]

\[
= \frac{3x+4x+12}{x+3} = \frac{7x + 3}{7} = x
\]

21. Graph \( f(x) = (x - 1)(x + 2)^2(x + 5)^3 \). Be sure to label all intercepts and determine the end behavior.

**Zeros**

- 1 \( \text{mult} \) 1 \( \Rightarrow \) \( \text{cross} \)
- 2 \( \text{mult} \) 2 \( \Rightarrow \) \( \text{touch} \)
- 5 \( \text{mult} \) 3 \( \Rightarrow \) \( \text{cross} \)

**End behavior**

\( f(0) = -500 \)
22. Graph \( g(x) = -2(x + 1)(x - 2)(x + 4) \). Be sure to label all intercepts and determine the end behavior.

\[
f(0) = -2(1)(-1)(4) = 16
\]

\text{Zeros:} -4, 1, 2, \text{ all with mult. 1 = does not cross}

23. Graph the rational function \( R(x) = \frac{2x - 4}{x + 1} \). Be sure to label all intercepts and asymptotes.

\[
R(x) = 2
\]

\[
= D \quad \frac{2x - 4}{x + 1}
\]

\[
= D \quad 2x - 4 = 2x + 2
\]

\[
= D \quad -4 = 2
\]

so the function never crosses the HA.

\[
R(0) = -4
\]

\[
R(-2) = \frac{-4 - 4}{-1} = 8
\]
24. Find the zeros of the function \( f(x) = 2x^3 + 9x^2 + 10x + 3 \).

Possible zeros: \( \pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2} \)

\[ f(1) = 2 + 9 + 10 + 3 \neq 0, \quad f(-1) = -2 + 9 - 10 + 3 = 0 \checkmark \]

so \( (x + 1) \) is a factor

\[
\begin{array}{c|ccr}
& 2 & 9 & 10 & 3 \\
\hline
-1 & 2 & -7 & -3 & 0 \\
\hline
2 & 7 & 3 & 0 & 0
\end{array}
\]

= 0

\[
f(x) = (x+1)(2x^2 + 7x + 3)
\]

\[= (x+1)(2x+1)(x+3)\]

\[\Rightarrow \text{zeros are: } -1, -\frac{1}{2}, -3\]

25. Graph \( f(x) = 1 + 8x - 2x^2 \) by completing the square i.e. get it into the form \( f(x) = a(x-h)^2 + k \)

and then transform \( g(x) = x^2 \). Be sure to label the vertex and any intercepts.

\[
f(x) = -2x^2 + 8x + 1 = -2(x^2 - 4x) + 1
\]

aside: \( \frac{-8}{2} = -4, (-2)^2 = 4 \)

\[= -2(x^2 - 4x + 4 - 4) + 1 \]

\[= -2(x^2 - 4x + 4) + 8 \Rightarrow f(x) = -2(x-2)^2 + 9 \]

\[\text{vertical shift: } f(x) \]

\[f(0) = 1\]

\[\text{intercepts: } -2x^2 + 8x = 0 \Rightarrow x = 0 \text{ or } x = 4\]

\[f(x) = -2(x-2)^2 + 9 \Rightarrow \frac{x-2}{\sqrt{2}} = \frac{9}{\sqrt{2}} \Rightarrow x = \frac{\pm \sqrt{18}}{\sqrt{2}} + 2 \]

\[x \text{- intercepts: } (0,0), (4,0)\]
26. If \( f(-2) = 1, f(-1) = 2, f(0) = 4 \) and \( f(1) = 8 \) find the exponential function that would generate the previous outputs. Sketch \( g(x) = f(x) - 1 \). Sketch \( f^{-1}(x) \).

Exponential = \( f(x) \) of the form \( f(x) = a^x \)

\[
\begin{align*}
f(-2) &= 1 = a^{-2} \\
f(-1) &= 2 = a^1 \\
f(0) &= 4 = a^2 \\
f(1) &= 8 = a^3
\end{align*}
\]

so our base is 2 and when we plug in -2 we get 1 so

\[
f(x) = 2^{x+3} \quad \text{and} \quad g(x) = 2^{x+3} - 1
\]

27. List the potential rational zeros of the function \( g(x) = 2x^3 + 4x^2 - 5 \).

\[
\pm 1, \pm 5, \pm \frac{5}{2}, \pm \frac{1}{2}
\]
28. Use synthetic division to determine whether the function \( f(x) = x^3 + 4x^2 - 4x - 16 \) has factors:

(a) \( x + 2 \)
(b) \( x - 3 \)
(c) \( x - 2 \)
(d) \( x + 4 \)

\[
\begin{array}{c|cccc}
\text{a)} & 1 & 4 & -4 & -16 \\
\text{2} & & 2 & -8 & 0 \\
\hline 
& 1 & 2 & -8 & 0 
\end{array}
\]

\( \Rightarrow x + 2 \) is a factor

\[
\begin{array}{c|cccc}
\text{b)} & 1 & 4 & -4 & -16 \\
\text{3} & 3 & 21 & 51 & 0 \\
\hline 
& 1 & 7 & 17 & 0 
\end{array}
\]

\( \Rightarrow x - 3 \) is not a factor

\[
\begin{array}{c|cccc}
\text{c)} & 1 & 4 & -4 & -16 \\
\text{2} & & 2 & 12 & 16 \\
\hline 
& 1 & 6 & 8 & 0 
\end{array}
\]

\( \Rightarrow x - 2 \) is a factor

\[
\begin{array}{c|cccc}
\text{d)} & 1 & 4 & -4 & -16 \\
\text{-4} & & -4 & 0 & 16 \\
\hline 
& 1 & 0 & -4 & 0 
\end{array}
\]

\( \Rightarrow x + 4 \) is a factor