1. Use the remainder theorem to find the remainder when \( f(x) = 8x^3 - 3x^2 + x + 4 \) is divided by \( g(x) = x - 1 \). Is \( g(x) \) a factor of \( f(x) \)?

\[
f(1) = 8 - 3 + 1 + 4 = 10
\]

\( = 0 \)  The remainder when \( f(x) \) is divided by \( g(x) \)  

\( = 0 \)  \( g(x) \) is not a factor of \( f(x) \)

2. List the potential rational zeros of the function \( f(x) = 12x^8 - x^7 + 6x^4 - x^3 + x - 3 \).

Possible numerators: \( \pm 1, \pm \frac{1}{3} \)

Possible denominators: \( \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12 \)

So possible rational zeros are: \( \pm 1, \pm \frac{1}{3}, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 1, \pm 2, \pm 3, \pm 6, \pm 12 \)

3. Find a polynomial of degree 7 with zeros \( 1, 2, 1 - \imath \) and \( 3 + 4\imath \).

\[
f(x) = (x-1)(x-2)(x-(1+i))(x-(1-i))(x-(3+4i))(x-(3-4i))
\]
4. Find the complex zeros of \( f(x) = x^3 - 3x^2 - 6x + 8 \).

Possible rational zeros: \( \pm 1, \pm 2, \pm 4, \pm 8 \)

\( f(1) = 0 \) so \( x-1 \) is a factor

\[
\begin{array}{cccc}
1 & -3 & -6 & 8 \\
\hline
1 & -2 & -8 \\
1 & -2 & -8 & 0
\end{array}
\]

\[ f(x) = (x-1)(x^2 - 2x - 8) \]

\[ = (x-1)(x-4)(x+2) \]

So zeros are \(-2, 1, 4\).

5. Find the zeros of \( f(x) = x^4 - 4x^3 + 9x^2 - 20x + 20 \).

Possible rational zeros: \( \pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20 \)

\( f(2) = 0 \) so \( x-2 \) is a factor

\[
\begin{array}{cccc}
1 & -4 & 9 & -20 \\
\hline
1 & -4 & 10 & -20 \\
1 & -2 & 5 & -10 & 0
\end{array}
\]

\[ f(x) = (x-2)(x^3 - 2x^2 - 3x + 10) \]

\[ = (x-2)(x^3 - 2x^2 - 3x + 10) \]

So zeros are: \( 2 \) with multiplicity 2

\[
\therefore f(x) = (x-2)^2(x+\sqrt{5})(x-\sqrt{5})
\]
6. Use synthetic division to determine whether the following factors divide \( f(x) = x^3 - 7x - 6 \):

(a) \( x + 1 \)

(b) \( x + 2 \)

(c) \( x - 2 \)

(d) \( x - 3 \)

\[ \begin{array}{c|cccc}
   & 1 & 0 & -7 & -6 \\
   -1 & & -1 & 1 & 6 \\
   \hline
   & 1 & -1 & -6 & 0 \\
\end{array} \]

\( \Rightarrow \) \( x + 1 \) is a factor

\[ \begin{array}{c|cccc}
   & 1 & 0 & -7 & -6 \\
   -2 & & -2 & 4 & 6 \\
   \hline
   & 1 & -2 & -3 & 0 \\
\end{array} \]

\( \Rightarrow \) \( x + 2 \) is a factor

\[ \begin{array}{c|cccc}
   & 1 & 0 & -7 & -6 \\
   2 & & 2 & 4 & -6 \\
   \hline
   & 1 & 2 & -3 & -12 \\
\end{array} \]

\( \Rightarrow \) \( x - 2 \) is not a factor

\[ \begin{array}{c|cccc}
   & 1 & 0 & -7 & -6 \\
   3 & & 3 & 9 & 6 \\
   \hline
   & 1 & 3 & +2 & 0 \\
\end{array} \]

\( \Rightarrow \) \( x - 3 \) is a factor
7. Use transformations to graph \( f(x) = 3^{-x} - 2 \). Be sure to label 3 points. What is the domain and range of the function? Where is the horizontal asymptote?

\[ \text{dom}(f) = \text{all reals} \]
\[ \text{range}(f) = (-\infty, 0) \]
\[ \text{hor. asympt. at } y = -2 \]

8. Use transformations to graph \( f(x) = -3^x - 2 \). Be sure to label 3 points. What is the domain and range of the function? Where is the horizontal asymptote?

\[ \text{dom}(f) = \text{all reals} \]
\[ \text{range}(f) = (-\infty, -2) \]
\[ \text{hor. asympt. at } y = -2 \]
9. Solve the following equations:

(a) \( e^x = e^{3x+8} \)
\[ x = 3x + 8 \Rightarrow -2x = 8 \Rightarrow x = -4 \]

(b) \( 5^{x^2+8} = 125^{2x} \)
\[ 5^{x^2+8} = (5^3)^{2x} \Rightarrow 5^{x^2+8} = 5^{6x} \Rightarrow x^2 + 8 = 6x \]
\[ x^2 - 6x + 8 = 0 \Rightarrow (x-2)(x-4) = 0 \Rightarrow x = 2, 4 \]

(c) \( 9^{-x+15} = 27^x \)
\[ 9^{-x+15} = (9^3)^x \Rightarrow 3^{-2x+45} = 3^{3x} \Rightarrow -2x + 45 = 3x \]
\[ -5x = -45 \Rightarrow x = 9 \]

(d) \( e^x = e^{3x} \cdot \frac{1}{e^2} \)
\[ e^x = e^{3x} \cdot e^{-2} \Rightarrow e^x = e^{3x-2} \Rightarrow x = 3x - 2 \]
\[ x^2 - 3x + 2 = 0 \Rightarrow (x-1)(x-2) = 0 \Rightarrow x = 1, 2 \]
10. Find the exact value of each logarithm:

(a) \( \log_{\sqrt{3}} 9 \)
(b) \( \log_{10} \sqrt{10} \)
(c) \( \ln \sqrt{e} \)
(d) \( \ln \sqrt{2^2} \)

\[
\begin{align*}
\text{a) } & \log_{\sqrt{3}} q = y \implies (\sqrt{3})^y = q \implies 3^{\frac{y}{2}} = q \implies \frac{y}{2} = 2 \implies y = 4 \\
\implies & \log_{\sqrt{3}} 9 = 4 \\
\text{b) } & \log_{10} \sqrt{10} = \log_{10} 10^\frac{1}{2} = \frac{1}{2} \\
\text{c) } & \ln \sqrt{e} = \ln e^{\frac{1}{2}} = \frac{1}{2} \\
\text{d) } & \ln \sqrt{2^2} = \ln 2^1 = 1
\end{align*}
\]

11. What is the domain of \( f(x) = \log_5 \left( \frac{x+1}{x} \right) \)?

When \( \frac{x+1}{x} > 0 \)

\[
\begin{array}{c|c|c|c}
\text{When is} & \left( -\infty, -1 \right) & (-1, 0) & (0, \infty) \\
\hline
\frac{x+1}{x} & + & - & + \\
\frac{x}{x} & - & 0 & + \\
\frac{1}{x} & 0 & - & + \\
\end{array}
\]

\( \text{dom}(f) = \left( -\infty, -1 \right) \cup (0, \infty) \)
12. What is the domain of $\sqrt{\ln(x)}$?

$$\text{dom} \left( \sqrt{\ln(x)} \right) = \left\{ x \mid x \text{ is in } \text{dom}(\ln(x)), \ln(x) \geq 0 \right\}$$

$$= \left\{ x \mid x > 0 \text{ and } \ln(x) \geq 0 \right\}$$

$$= \left\{ x \mid x > 0 \text{ and } x \geq 1 \right\}$$

$$= \left\{ x \mid x \geq 1 \right\}$$

13. Sketch the graph of $f(x) = \log(-x) + 1$. Label 3 points and any asymptotes.

Note: $\log(x) = \log_{10}(x)$

![Graph of $f(x) = \log(-x) + 1$ with labeled points and asymptotes]
14. Solve the following equations:

(a) \( \log_5 x = 3 \)

(b) \( \ln e^2 = 5 \)

(c) \( \log_6 36 = 5x + 3 \)

(d) \( e^{2x} = 10 \)

(e) \( \log_2 \left( \frac{1}{8} \right) = 3 \)

\[ \begin{align*}
\text{(a)} & \quad \log_5 x = 3 & \Rightarrow & \quad 5^3 = x & \Rightarrow & \quad 125 = x \\
\text{(b)} & \quad \ln e^2 = 5 & \Rightarrow & \quad \ln e^2 = e^5 & \Rightarrow & \quad 5 = e^5 \\
\text{(c)} & \quad \log_6 36 = 5x + 3 & \Rightarrow & \quad \log_6 6^3 = 5x + 3 & \Rightarrow & \quad 3 = 5x + 3 \\
& & & & & \\
& & & & & \\
\text{(d)} & \quad e^{2x} = 10 & \Rightarrow & \quad \ln (e^{2x}) = \ln (10) & \Rightarrow & \quad 2x = \ln (10) \Rightarrow x = \frac{\ln (10)}{2} \\
\text{(e)} & \quad \log_2 \left( \frac{1}{8} \right) = 3 & \Rightarrow & \quad x^3 = \left( \frac{1}{8} \right)^6 & \Rightarrow & \quad x = \sqrt[3]{\frac{1}{8}} = \frac{1}{2} \\
\end{align*} \]
15. If \( \ln 2 = a \) and \( \ln 3 = b \) use properties of logs to write each logarithm in terms of \( a \) and \( b \).
   
   (a) \( \ln 8 \)
   
   (b) \( \ln \sqrt[3]{6} \)

\begin{align*}
   \ln 8 &= \ln 2^3 = 3 \ln 2 = 3a \\
   \ln \sqrt[3]{6} &= \frac{1}{3} \ln (2 \cdot 3) = \frac{1}{3} (\ln 2 + \ln 3) = \frac{1}{3} (a + b)
\end{align*}

16. Write the following logarithm as the sum and difference of logarithms and write powers as factors:

\[
f(x) = \ln \left[ \frac{5x^2 \sqrt{1-x}}{4(x+4)^2} \right], \quad 0 < x < 1.
\]

\[
= \ln (5x^2 (1-x)^{\frac{1}{2}}) - \ln 4(x+4)^2
\]

\[
= \ln 5 + \ln (x^2 (1-x)^{\frac{1}{2}}) - \ln 4 - \ln (x+4)^2
\]

\[
= \ln 5 + \ln x^2 + \ln (1-x)^{\frac{1}{2}} - \ln 4 - \ln (x+4)^2
\]

\[
= \ln 5 + 2 \ln x + \frac{1}{2} \ln (1-x) - \ln 4 - 2 \ln (x+4)
\]

17. Write the following as a single logarithm: \( 3 \log(3x+1) - \log(x^2 - 1) + 2 \log(x+1) \). Simplify as much as possible.

\[
3 \log(3x+1) - \log(x^2 - 1) + 2 \log(x+1) = \log \left( (3x+1)^3 \right) - \log(x^2 - 1) + \log(x+1)^2
\]

\[
= \log \left( \frac{(3x+1)^3}{x^2 - 1} \right) + \log(x+1)^2 = \log \left( \frac{(3x+1)^3}{(x+1)(x-1)} \right) = \log \left( \frac{(3x+1)^3(x+1)}{x-1} \right)
\]
18. Solve the following logarithmic expressions:

(a) \(3 \log_2(x - 1) + \log_2 4 = 4\)
(b) \(\log_2(x + 7) + \log_2(x + 8) = 1\)
(c) \(\log_8(x + 6) = 1 - \log_8(x + 4)\)

19. Solve \(3^{1-2x} = 4^x\).

\[
= \ln \left( 3^{1-2x} \right) = \ln (4^x) \\
= (1-2x)\ln(3) = x\ln(4) \\
= \ln(3) - (2\ln(3))x = x\ln(4) \\
= -2x\ln(3) = x\ln(4) - \ln(3) \\
= x \left( -2\ln(3) - \ln(4) \right) = -\ln(3) \\
= x = \frac{\ln(3)}{2\ln(3) + \ln(4)}
\]
20. Find the amount that results from each investment:

(a) $50 invested at 6% compounded monthly after a period of 3 years.
(b) $600 invested at 5% compounded daily after a period of 3 years.

\( A = P \left(1 + \frac{r}{n}\right)^{nt} \)

(a) \( P = 50 \)
\( r = 0.06 \)
\( n = 12 \)
\( t = 3 \)

\( A = 50 \left(1 + \frac{0.06}{12}\right)^{12 \times 3} \approx 59.83 \)

(b) \( P = 600 \)
\( r = 0.05 \)
\( n = 365 \)
\( t = 3 \)

\( A = 600 \left(1 + \frac{0.05}{365}\right)^{365 \times 3} \approx 697.09 \)

21. Find the principal needed today to get each new amount.

(a) To get $75 after 3 years at 8% compounded quarterly.
(b) To get $600 after 2 years at 4% compounded quarterly.

\( A = 75 \)
\( t = 3 \)
\( r = 0.08 \)
\( n = 4 \)

\( 75 = P \left(1 + \frac{0.08}{4}\right)^{4 \times 3} \Rightarrow P = \frac{75}{\left(1 + \frac{0.08}{4}\right)^{4 \times 3}} \approx 59.14 \)

(a) \( A = 600 \)
\( t = 2 \)
\( r = 0.04 \)
\( n = 4 \)

\( 600 = P \left(1 + \frac{0.04}{4}\right)^{4 \times 2} \Rightarrow P = \frac{600}{\left(1 + \frac{0.04}{4}\right)^{4 \times 2}} \approx 594.09 \)
22. On January 1, Kayla places $1000 in a COD that pays 6.8% compounded continuously and matures in 3 months. She then takes her $1000 and the interest earned in a savings account that pays 5.25% compounded continuously. How much does Kayla have in her savings account on May 1?

\[ P = 1000, \quad r = 0.068, \quad t = \frac{3}{12} \text{ year} \]
\[ A_1 = 1000 e^{(0.068) \frac{3}{12}} \approx \$1017.15 \]

After 3 months it is April 1st, we take \( A_1 \) and put it into our new account.

\[ P = A_1, \quad r = 0.0525, \quad t = \frac{1}{12} \text{ year} \]
\[ A_2 = A_1 e^{(0.0525) \frac{1}{12}} \approx \$1021.61 \]

23. The number \( N \) of bacteria present in a culture at time \( t \) (in hours) obeys the law of exponential growth \( N(t) = 1000e^{0.01t} \).

(a) How many bacteria are present at time \( t = 0 \)?
(b) What is the growth rate of the bacteria?
(c) What is the population after 4 hours?
(d) When will the number of bacteria reach 1700?
(e) When will the number of bacteria double?

\[ N(0) = 1000 \]
\[ \text{growth rate} = 0.01 \]
\[ N(4) = 1000 e^{0.04} \approx 1041 \]
\[ 1700 = 1000 e^{0.01t} \Rightarrow 1.7 = e^{0.01t} \Rightarrow \ln(1.7) = \ln(e^{0.01t}) \]
\[ \Rightarrow \ln(1.7) = 0.01t \Rightarrow t = \frac{\ln(1.7)}{0.01} \approx 53 \text{ hrs} \]
\[ 2000 = 1000 e^{0.01t} \Rightarrow 2 = e^{0.01t} \Rightarrow \ln(2) = 0.01t \]
\[ \Rightarrow t = \frac{\ln(2)}{0.01} \approx 69.3 \text{ hrs} \]
24. The half-life of radium is 1690 years. If 10 grams are present now, how much will be present in 50 years?

\[ N(t) = N_0 e^{kt} \]

Half-life is 1690 years, so \[ N(1690) = \frac{1}{2} N_0 = N_0 e^{k \cdot 1690} \]

\[ e^\frac{1}{2} = e^{k \cdot 1690} \]

\[ \ln(e^{\frac{1}{2}}) = \ln(e^{k \cdot 1690}) = k \]

\[ \ln(1) - \ln(\frac{1}{2}) = -\ln(2) = k \]

This means our general equation is \[ N(t) = 10 \cdot e^{(-\ln(2)) \cdot \frac{t}{1690}} \]

Thus, after 50 years we have

\[ N(50) = 10 \cdot e^{(-\ln(2)) \cdot \frac{50}{1690}} \approx 9.797 \text{ grams} \]