Answer all questions to the best of your ability. Full credit will only be given if all work is shown and organized and it is clear what your answer is. The review is out of 25 points.

1. The kinetic energy of a moving object varies jointly with its mass $m$ and the square of its velocity $v$. If an object weighing 25 kilograms and moving with a velocity of 10 meters per second has a kinetic energy of 1250 joules, find its kinetic energy when the velocity is 15 meters per second.

\[ E = \frac{1}{2}mv^2 \]

\[ m = 25 \]
\[ v = 10 \]
\[ E = 1250 \]

\[ E = \frac{1}{2} \times 25 \times (15)^2 = \frac{5262.5}{2} \text{ joules} \]

2. The cube of $z$ varies directly with the sum of the squares of $x$ and $y$. Find the general equation if $z = 2$ when $x = 9$ and $y = 4$.

\[ z^3 = k(x^2 + y^2) \]

\[ z = 2, \ x = 9, \ y = 4 \]

\[ 8 = k(81 + 16) \]

\[ k = \frac{8}{97} \]

so general equation is:

\[ z^3 = \frac{8}{97}(x^2 + y^2) \]

3. Answer the following questions about the relation \{ (0,1), (1,1), (2,1), (3,1), (4,4) \}.

(a) Is the relation a function? Why or why not?  Yes, each input has exactly one output.

(b) What is the domain of the relation?  \{ 0, 1, 2, 3, 4, 5 \}

(c) What is the range of the relation?  \{ 1, 4, 5 \}
4. What is the domain of the function \( \frac{\sqrt{x^2 + 1}(3 - x)}{2x^2 - 4x - 6} \)?

\[
\begin{align*}
\frac{f(x)}{x^2 + 1} &= \frac{\sqrt{x^2 + 1} - x}{2x^2 - 4x - 6} = \frac{\sqrt{x^2 + 1} (3 - x)}{2(x^2 - 3x - 3)} = \frac{\sqrt{x^2 + 1} (3 - x)}{2(x - 3)(x + 1)}
\end{align*}
\]

\[x^2 + 1 \geq 0 \quad \text{for all } x\]

\[\Rightarrow \quad \text{dom}(f) = \{x \mid x \neq 3, x \neq -1\} \]

5. If \( f(x) = 2x^2 + 2 \) and \( g(x) = 3x - 2 \) what are (simplify):

(a) \( (f + g)(x) = f(x) + g(x) = 2x^2 + 2 + 3x - 2 = 2x^2 + 3x \quad \text{with domain } \mathbb{R} \)

(b) \( (f - g)(x) = f(x) - g(x) = 2x^2 + 2 - 3x + 2 = 2x^2 - 3x + 4 \)

(c) \( (f \cdot g)(x) = f(x)g(x) = (2x^2 + 2)(3x - 2) = 6x^3 + 6x - 4x^2 - 4 = 2(3x^2 + 2x^2 + 6x - 2) \)

(d) \( \left( \frac{f}{g} \right)(x) = \frac{f(x)}{g(x)} = \frac{2x^2 + 2}{3x^2 - 2} \quad \text{with domain } \mathbb{R} \setminus \{x \mid x \neq \frac{2}{3}\} \)

(e) \( (f + g)(1) = f(1) + g(1) = 5 \)

(f) \( (f - g)(0) = f(0) - g(0) + 2 = 4 \)

(g) \( (f \cdot g)(2) = f(2)g(2) = (2(3)(2)(2) + 3(2) - 2) = 2(34 - 8 + 6 - 2) = 40 \)

(h) \( \left( \frac{f}{g} \right)(-1) = \frac{f(-1)}{g(-1)} = \frac{4}{-5} \)

(a) \( \text{dom}(f) = \text{range}(f) = \text{all real } x \)
(b) inc. on \( (-\infty, -3) \cup (3, \infty) \)
(c) dec. on \( (-3, 3) \)
(d) local max at \( x = -3 \) with value \( 1 \)
(e) local min at \( x = 3 \) with value \( -1 \)
(f) odd
(g) y-int at \( y = 0 \)
(h) x-int at \( x = -3, 0, 3 \)

7. Are the following functions even, odd or neither?

(a) \( f(x) = x^3 - 4x \)
\[ f(-x) = (-x)^3 - 4(-x) = -x^3 + 4x = -f(x) \]
Even

(b) \( g(x) = \frac{4 + x^2}{1 + x^4} \)
\[ g(-x) = \frac{4 + (-x)^2}{1 + (-x)^4} = \frac{4 + x^2}{1 + x^4} = g(x) \]
Even

(c) \( H(x) = 1 + x + x^2 \)
\[ H(-x) = 1 + (-x) + (-x)^2 = 1 - x + x^2 \]
Neither

8. Find the average rate of change from 2 to 3 of the function \( f(x) = 2x^2 + 7 \). Then find the equation of the secant line between (2, \( f(2) \)) and (3, \( f(3) \)).

\[ f(3) = 2(3)^2 + 7 = 25 \]
\[ f(2) = 2(2)^2 + 7 = 11 \]

\[ \text{Ave. rate of change} = \frac{25 - 11}{3 - 2} = 14 \]

\[ \text{Slope of secant line} \]

\[ y - 11 = 14(x - 2) \]
9. Graph each of the following functions using translations and reflections. Identify at least 2 points on the final graph. State the domain and, based on the graph, find the range. Be sure to show a graph of your original function and how the graph changes at each step.

(a) \( f(x) = |x| - 4 \)

(b) \( F(x) = -\sqrt{x + 3} \)

(c) \( g(x) = -(x - 1)^3 - 1 \) (Hint: \( g(x) = -[(x - 1)^3 + 1] \))
10. Answer the following questions about the piecewise function \( f(x) = \begin{cases} 
  x & \text{if } -4 \leq x < 0 \\
  1 & \text{if } x = 0 \\
  3x & \text{if } x > 0 
\end{cases} \)

(a) What is the domain of the function?
\[ \text{dom}(f) = [-4, \infty) \]

(b) Locate any intercepts.
\( \gamma\text{-int:} \) \( f(0) = 1 \)
\( x\text{-int:} \) \( \text{none} \)

(c) Graph the function.

(d) Based on the graph find the range.
\[ \text{range}(f) = [-4, 0) \cup (0, \infty) \]

(e) Is \( f \) continuous on its domain? Why or why not?
\( \text{not continuous at } x = 0 \), \( \text{it has a hole} \).
11. Answer the following questions about the piecewise function \( f(x) = \begin{cases} \ x^2 & \text{if } -2 \leq x \leq 2 \\ 2x - 1 & \text{if } x > 2 \end{cases} \)

(a) What is the domain of the function?
\( \text{dom}(f) = [-2, \infty) \)

(b) Locate any intercepts.
\( y \)-int: \( f(0) = 0 \)
\( x \)-int: \( f(x) = 0 \) when \( x = 0 \)

(c) Graph the function.

(d) Based on the graph find the range.
\( \text{range}(f) = [0, \infty) \)

(e) Is \( f \) continuous on its domain? Why or why not?
no, there is a gap when \( x = 2 \).